DG 10: Different perspectives, positions, and approaches in mathematics education research

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Aims and focus

The main issue discussed in the group was the difficulty of accumulating knowledge in mathematics education research (m.e.r. for short) in view of the existence of a diversity of approaches to mathematics education research that sometimes appear as passing fads or fashion waves. On the one hand, the diversity could be seen as an advantage because it promises a more complete picture. On the other, it causes fragmentation, which could be an obstacle to recognizing m.e.r. as a discipline, characterized by a coherent body of knowledge. Moreover, the diversity complicates communication: giving examples of concrete results is difficult without a lengthy presentation of the theoretical underpinnings. The fashion waves in m.e.r. also have their advantages; by focusing attention on a single aspect, they allow this aspect to be thoroughly examined. Too often, however, when the fashion fades, the deep results obtained during this period are forgotten. There is a risk of lack of real progress, and of missing the chance of laying a strong and lasting foundation of research for understanding of educational phenomena.

Written contributions

Three papers were accepted for distribution and publication on the web:
David Clarke, "Issues of voice and variation: The problematics of international comparative research in mathematics education" (Australia).
Bettina Dahl, "Can different theories of learning work together? Some results from an investigation into pupils' metacognition" (Norway).
Steve Lerman and Anna Tsatsaroni, "Surveying the field of mathematics education research".

The organization of the sessions

The first session started with an introduction to the theme of the discussion group and continued with a panel discussion. The panelists were: Gerald A. Goldin, Marie-Jeanne Perrin-Glorian, Lyn English, Anna Sierpinska and Tatyana Oleinik.

The aim of the panel discussion was to provide examples of approaches, theories, concepts that made particularly brilliant careers in mathematics education, and which have later been criticized and either abandoned or, on the contrary, transformed and developed. We decided that panelists were going to be much more convincing if they spoke from their own experience.

The second session was devoted to discussing, in two subgroups, the following two questions:

1. What can research in mathematics education tell us about the constraints that define the reality of teaching and learning mathematics and limit our possibilities of changing this reality, independently of the "approach" used in this research?

2. What are the possible research questions, methodologies and anticipated research results that stem from our answers to question 1?

One subgroup (15 people) considered the questions from the "macro-level" of curriculum studies, educational policy issues, research priorities issues, international studies of scholastic achievement, cross-cultural studies, and the "meso-level" of the organization of teaching of a particular mathematical content by a teacher in a classroom; design and evaluation of teaching materials and learning environments. The other group (33) concentrated on the "micro level" of studies of particular classroom interactions, learning difficulties related to a specific mathematical content, evaluation of teaching experiments, etc.

The third session started with presentations of summaries of discussions in the subgroups and continued with a plenary discussion of the issues raised by the group.

Some details of the discussions

Summaries of the panel contributions

Three panelists confirmed the experience of "fashions" in their lives and two others denied it by seeing the development more as expansion and building on the previous results.

G.A. Goldin: 'I spoke to the history of "paradigms", "fashions", and all-encompassing claims in mathematics education research from the 1960s to the present time. One example (which influenced me in my early work, but in my view proved insufficient) was the emphasis in the 1970s and 1980s on "artificial intelligence" models – the human thinker as essentially an information processing system, with computer simulations of human thinking processes as fundamental to the paradigm. A second example was the ascendance of behaviorism – today it is difficult to appreciate how predominant behaviorist ideas became in the early 1970s, as a reaction to the "new math". A third example was the dominance of radical constructivism during the1990s, from which our field is only now shaking loose. I addressed the need for a synthetic and eclectic approach that includes rather than excludes the many different, important constructs that have previously been viewed as mutually exclusive.'

L. English: English described how her early research career was strongly influenced by the cognitive movements of the 1980s. She referred to the computer metaphor for learning as a basis of her research on mathematical problem solving. Her work focused on the nature and types of knowledge, on higher-order thinking processes, and on the interactions between knowledge forms and thinking processes. There was also an emphasis on individual learning, with detailed analyses of individual children's mathematical reasoning during problem solving. No consideration was given to environmental issues in students' learning. In contrast, English's research now focuses on both the cognitive and social aspects of children's mathematical learning, together with the professional development of their teachers. Her analyses of learning include children's developments during collaborative problem-solving situations, with a focus on their mathematical modeling. The mathematical growth of their teachers (both content and pedagogy) is also a strong component of her research today.

A. Sierpinska: 'In my life as mathematics educator I have known at least these 'fashions' or, rather, focalizations in m.e.r.: focus on *mathematical theory*: pedagogical

organization of mathematical material; – *mathematical epistemology*: epistemological studies of mathematical concepts, in abstraction from the socio-cultural conditions of learning at school; *Constructivism*: study of students' construction of mathematical concepts; *The body in the mind*, including *Instrumentation*: taking into account the physical body of the learner and his/her interaction with tools in learning mathematics. I have also witnessed the rise of some words as key words for research and then the ban of these same words as representing a backward philosophy (e.g. misconception, obstacle, understanding, reality). The rise and fall of "epistemological obstacle" is part of my personal story. There was a time in mathematics education when this was a fashionable concept, with special publications and conferences. This concept refined the notion of error in mathematics, turned it into something serious, and changed our attitude to students' errors. And then, in the mid-90s, from the post-modern perspective, epistemological obstacles became a bad word; the philosophy underlying epistemological obstacles was criticized for being "recapitulationistic and parallelistic", for not sufficiently taking into account the socio-cultural factors.'

T. Oleinik: Tatyana referred to her poster, co-authored with Victor Yevdokimov, about the traditional methods of conducting educational research in Ukraine. These methods favor a pluralistic approach, which is not considered as a sequence of passing fads, but as a way of taking into account as many factors influencing teaching and learning as possible in the design, implementation and evaluation of teaching approaches.

M.J. Perrin-Glorian: 'Change and continuity needn't contradict each other. As a researcher within the French school of didactics of mathematics, I see my own story as a process of enrichment of theories to take into account more of the classroom complexity. The research project remains the same: to find and study the conditions for the best possible mathematics teaching. I see three stages in my research history.

1975-1984*:* Design, implementation and analysis of teaching, based on initial versions of the Theory of Didactic Situations (Brousseau, 1997). and the theory of the Tool-Object Dialectics (Douady, 1987). The focus was on epistemology of mathematics, and the main problems were: To what extent can mathematical situations themselves trigger knowledge construction in students, thus reproducing the conditions of production of original mathematical knowledge?

1984-1993*:* Difficulties in implementing, in classes of low achievers, situations that worked well in other classes, gave rise to new questions and require complementary theoretical elements. To explain the discrepancy between the planned situations and what actually happened in the classes, I first used the theoretical frame of "metacognitive representations", concerning students' and teachers' ideas about mathematics and mathematical teaching At the same time, I was becoming acquainted with the developments in the anthropological theory of didactics, which was attempting to connect the micro and macro levels of didactic analyses and theories. The distinction as well as links between institutional and personal relationships to knowledge that this theory introduced, became, for me, a way to transform the notion of social representations in a way that was better adapted to didactical questions and more coherent with other theoretical choices. From my research in low achievers' classes, I was now convinced that a good "didactic transposition" of knowledge for the purposes of its teaching was not enough: trying to improve teaching by taking into account only epistemology and student's difficulties may produce worse learning than traditional teaching. Teachers' resistance to new practices, the overlapping of students' difficulties and teachers' choices showed that

it was not a question of personal representations. Theory was needed to address questions such as: Do teachers have any choices? What are their choices?

Since 1993: I used the theory of didactical situations mainly to produce good teaching situations. But further developments of this theory, complemented by elements of the anthropological theory, allowed to apply it to the study of ordinary mathematics classes.

Thus, from my experience, research questions changed while theoretical frames were growing. I think that three reasons explain, in the French context, how theoretical construction preserves and enriches previous elements instead of replacing them: the will to construct a specific coherent theoretical framework, theoretical options allowing this coherence, and the existence of institutions supporting continuous exchanges and debates between researchers.'

Notes about the group discussions

As could be expected, the subgroup discussions and even the final plenary discussion diverged somewhat from the questions initially posed. There was a tendency to question the questions themselves. Also, expressions such as "conceptual change" and "evolution" were seen as better describing developments in m.e.r. than "progress" and "accumulation".

In the subgroup supposed to concentrate on the "micro level", the focus was on achievements of m.e.r.: participants were asked to name a result that surprised them personally and had a significant impact on their research or teaching practice. Many results were mentioned but, generally, evolution was seen in research which helped us better understand the boundaries of our freedom to change the reality of teaching mathematics according to a prevailing ideology of the time. Examples included problematization and study of aspects long taken for granted in the teaching of mathematics, apparent in concepts such as, e.g. "socio-mathematical norms", "Zone of Proximal Development", or "didactic transposition". Evolution was seen in an increased awareness of the differences between students' ways of knowing and the ways of knowing that teachers expect them to develop; and in the greater acknowledgement of the need to develop content specific didactic means to bridge the gap without eradicating these individual ways of knowing which are seen as potentially creative and fruitful. If anything, research on students' conceptions has overthrown the naïve belief that if only the teacher used the right words in explaining concepts, everybody would understand.

In the sub-group that addressed "macro-issues", five questions were highlighted for consideration:

1) The kinds of questions and problems addressed in mathematics education research.
2) The objects of this research.
3) The issues associated with different schools of thought.
4) Issues pertaining to methodology.
5) The driving forces behind mathematics education research movements.

This group commenced discussion with the question, "Why is there an attack on previous theories, methodologies, and movements?" In addressing this issue, comparisons were made between research in education and research in medicine, where it was pointed

out that education tries to solve problems for the present time, whereas medical research attempts to solve problems for both now and the future. Furthermore, medical research builds on existing research; the extent to which education adopts this course of action was debated. Other points raised in addressing this question included: (a) there appear to be social, political, and academic rewards for attacking previous movements; and (b) the education research community does not have a common knowledge base on which to refute some of the extravagant claims made (it was noted that "extravagant claims require extravagant evidence"). It was concluded that, as a community, we are not giving adequate attention to knowledge accumulation; we have not been sufficiently willing to say which ideas, theories, studies, etc., are important. Our problem is not that we use different ways of proving the claims we make, but rather that we don't use them.

In addressing issues pertaining to methodology, the group agreed that we need to use all methodologies: qualitative and quantitative research answer different questions. The swing back to quantitative research methodologies was considered a real concern to our community. It was raised that one common yardstick for "measuring" students mathematical achievement is to present tasks that enable them to display their achievements, in contrast to performance based studies that highlight failure.

With regard to "objects of study", the group discussed the use of the mathematical construct as the object of study, rather than the student, classroom, teacher etc. In focusing on the mathematical construct, consideration could be given to: (a) how it has developed historically, (b) how the student understands it, and (c) how the teacher understands it. *However*, the point was made that, oftentimes, it is not clear what the object of study is.

In addressing issues pertaining to "schools of thought," the group considered there were serious impediments to creating synthesis in mathematics education research. It was noted that we need time to find agreement between the various schools of thought (which have been presented as opposing ideas, such as the socio-cultural perspective overtaking and ignoring the cognitive aspects). Different schools of thought have different kinds of roles, yet researchers have tended to use them as a platform for justifying their own approach (oftentimes, however, the school of thought doesn't support the researcher's study). The group agreed that we need to respect different schools of thought for what each has to offer and take the best of each one. It was also stressed that mathematics educators need to understand the *entire* school of thought, not just aspects of it.

Finally, the group concluded that there is a need for more theoretical development in our discipline; *but* the question of balance in theoretical perspectives was emphasized. The question of "what the next ideology will be" was raised as a means of warning us not to ignore multiple perspectives on mathematics learning and teaching.

References

Brousseau, G.: 1997, *Theory of Didactical Situations in Mathematics*, Dortrecht: Kluwer Academic Publishers.

Douady, R.: 1987, 'Jeux de cadres et dialectique outil-objet', *Recherches en Didactique des Mathématiques* **7** (2), 5-31.

Jere Confrey was not able to come to the congress. Lyn Lyn English and Anna Sierpinska led the group sessions at the congress and compiled this report. They are happy to be contacted at l.english@qut.edu.au and sierpan@alcor.concordia.ca for further information on the work of this DG.