DG 19: Current problems and challenges in lower secondary mathematics education

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Aims and focus

The following issues were selected by the organizing team:

1. *Mathematical literacy and "mathematics for everybody"*
How do we define "mathematics for everybody"? Is this what we could name a "minimum curriculum"?
- Does it just include applied mathematics?
- How can teachers foster students' ability to apply mathematics skills to different contexts?

2. *Relationships between different levels of knowledge*
In a constructivist approach, mathematics should be taught through activities that invite pupils to reason, explain and justify rather than simply to memorize and imitate, in order to construct mathematical understanding. Nevertheless, memorizing and imitating are parts of the learning process.
- Is it possible to find a "right balance"?
- What are the relationships between computational skills and reasoning or understanding?

3. *Different approaches to geometry*
- What kinds of geometrical reasoning do 11-15 -year- old pupils develop?
- How will dynamic geometry software (for example *Cabri*) change the teaching of geometry?
- Are there important differences between countries in the way geometry is taught? (Inductive and deductive reasoning, modelling, application of software)

4. *What is the role of algebra in lower secondary school?*
- Should algebra be taught to all students? What aspects of algebra are of value to everyone? What should a minimum curriculum consist of? How do answers to these questions relate to regional or cultural differences?
- What do we expect of an algebra-literate individual? What are the values of algebra learning for the individual, especially in view of increasingly powerful computing capabilities offered by ICT systems?
- How can we reshape the algebra curriculum so as to have more immediate value to individuals? Can we identify explicit examples in contexts meaningful to students in which algebraic ideas have a clear and unambiguous value? Are there undesirable consequences of such orientations to algebra?

5. *The role of technology and electronic tools*

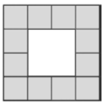
- How can the use of calculators and different software facilitate or – on the contrary – disturb mathematical learning? In which ways should such tools be used?
- Does the use of computers induce changes in curricula?

The two main challenges at the lower secondary level in many countries are the introduction of algebra and of deductive geometry. Therefore these topics were chosen as the main entries for the first and second 2-hour sessions.

First session

The question of mathematical literacy, mathematics for everybody, was posed as an introduction to the first session, followed by a workshop about the learning of algebra.

In order to launch the discussion on algebra, three tasks had been proposed for analysis. The first was the well known situation 'the border':



How to predict how many square tiles will be used to border any square?

This task represents an approach to algebra through its generalization function. It was pointed out that the problem may be set at different ages and levels, using material squares or letter symbols. Several algebraic expressions may be found by pupils, introducing the notion of equivalent expressions. The second task, entitled "tricks", has quite similar aims, although it is also connected with equations. It presents phenomena that can be explained by properties of operations and mastered by the use of letter symbols. For example:

Choose a decimal number
Calculate its double and its triple
Add the two results
Divide the result by 10
Given the final result, is it easy to guess the initial number?

The discussion centred on how this task could be used in the classroom. For example, after solving the task, pupils could be asked to create similar "tricks" and describe them by formulas.

A third task aimed at introducing the notion of equation.

Some participants were surprised to learn that the three tasks would be set at the same level in French classrooms (second year of secondary school, 12-13 year olds). The age for introduction of equations is different from one country to another, and this seems to be linked to different approaches to algebra. When generalization or functional perspectives are chosen for the introduction of algebra, the teaching of equations may occur later in the curriculum.

For lower secondary school, comparisons between countries are not easy because of the variety of conditions: lower secondary school may last from 3 to 5 years, the age of pupils ranges from 11 to 16, with different structures, one or several curricula, etc. There are countries where solving equations is not a part of compulsory teaching.

Second session

Time was shared between two topics: geometry and role of technology.

Maryvonne Le Berre, in a short overview, presented the problems linked to the introduction of deductive geometry, the relationships and opposition between practical geometry and deductive geometry, argumentation and proof, passing from drawings to figures, and the role of mathematization.

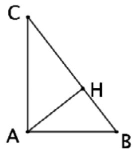
This was followed by a discussion about practices in proving the pythagorean theorem by means of the following questions:

> *Are the pupils given a mathematical proof, several proofs or only a demonstration by visualisation?*
> *What kind(s) of proofs are used more often?*
> *What are the reasons which lead to choosing one proof over another?*

In the discussion the classification given by Christine Knipping [4] was used:

1) Proofs based on comparisons of areas,
2) Proofs based on calculation of areas,
3) Proofs applying theorems of similarity, like the following

$AB^2 = HB \times BC$
$AC^2 = HC \times BC$
Hence
$AB^2 + AC^2 = (HB + HC) \times BC = BC^2$



4) Proofs using visualisation of Euclid's proof, i.e. $a^2 = pc$ and $b^2 = qc$ (Turning squares into rectangles).

The four kinds of proofs seemed to be equally frequently used amongst the represented countries, but comparison of areas is more often used to present and explain the theorem (what is expressed in English by the word "demonstration") than for proving (in French "démonstration"). In contrast, in French instruction, this kind of proof may be chosen for the reason that pupils are able to master each step of the proof, one of the aims being to teach them how to prove statements. (Knipping [4] [5])

Some participants noted that pupils can't find any proof of the pythagorean theorem by themselves, and asked "Is this a good situation for proof?" A cultural perspective may lead to using different kinds of proofs in the classroom.

Finally, the discussion focused on *the role of proof in geometry. Again, t*he role of proof is different in different countries. Most people focus on understanding. Believing, knowing and proving are different levels that ought to be distinguished by pupils, but "knowing" is sometimes considered as sufficient.

In the second part of this session Merrilyn Goos introduced a set of issues on the role of technology of which the following were discussed.

Pedagogical issues:

- For what purposes is technology used?
- How does use of technology help or hinder students' learning?
- What teaching approaches are effective?

- How can tasks be developed that engage students with significant mathematical concepts?

Curriculum issues:
- How might technology change the content of curricula?
- What should be omitted? What should be added?
- What criteria should be used in making these decisions?
- Which students (and courses) should have access to technology?

Assessment issues:
- How can assessment be designed so as to recognise and test students' learning when technology is present?
- What is the role of 'technology-free' assessment tasks?

Participants responded with a range of comments and additional questions: For most curricula, it does not seem to be a case of "all calculator" or "no calculator". It seems to be fruitful to consider responsible uses of calculators. Students need some "no calculator" experience first. The question is: How much? Who decides what to leave out? Often it is people who already understand mathematics and learned it without calculators. What should be omitted? Using logarithm and trigonometry tables! What could be added? Studying iterative processes, complex experimental problems, exponential growth and suchlike! How might pedagogy change when the children have grown up with technology and computers in their environment? What learning and what teaching could come from electronic games? How can teachers capitalize on the abilities that pupils put to use in these games?

Third session

In the last one-hour session, there was a spontaneous discussion, from the definition of mathematical literacy given in OECD/PISA (OECD, 2003, p. 20):
Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays to the world, to make well founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

Each participant was invited to propose answers to the question: *What should we expect of a mathematically literate person?* Responses included: Students should be problems solvers, and this involves communicating, reasoning, making connections, being creative. Students should be able to analyse situations, select data, pose mathematical questions from them, make investigations, evaluate, ask by themselves "what if?". Students should understand the news, the market place, measurement, basic probabilities. Affective issues are also important – what about enjoyment, confidence, perseverance, challenge?

Conclusion

In different countries, lower secondary school mathematics faces very different realities. This makes it sometimes difficult to identify and compare instructional choices.

In future ICMEs it might be relevant to create two discussion groups, one around the relation between primary and secondary school concerning pupils under the age of 13, and one discussing the relation between lower and upper secondary levels.

References

OECD (2003) *The PISA 2003 Assessment Framework – Mathematics, Reading, Science and Problem Solving Knowledge and Skills.* Paris: OECD.

The following papers are available from the ICME-10 website

[1] Learning from comparing. A review and reflection on qualitative oriented comparisons of teaching and learning mathematics in different countries, *Christine Knipping, University of Hamburg, Germany*

[2] Mathematics for Everybody; implications for the lower secondary school, *Steve Thornton University of Canberra, John Hogan, Redgum Consulting Pty Ltd., Australia*

[3] The differences between design intentions and implementation: The implementation of the Malaysian mathematics curriculum, *Noor Azlan Ahmad Zanzali, Faculty of Education, Universiti Teknologi, Malaysia*

[4] Towards a comparative analysis of proof, *Christine Knipping, University of Hamburg, Germany*

[5] Argumentations in proving discourses in mathematics classrooms, *Christine Knipping, University of Hamburg, Germany*

[6] How to deal with Algebraic Skills in Realistic Mathematics Education? *Monica M. Wijers, Freudenthal Institute, The Netherlands*

[7] Literal calculation and equations during French collège time, *Maryvonne Le Berre, IREM de Lyon, France*

[8] Misconceptions in Mathematics: Solving the Equation, *Suwattana Eamoraphan, Faculty of Education, Ghulalongkom University, Bangkok, Thailand*

[9] Affects and beliefs in school mathematics: gender differences, *Gard Brekke, Lise Wiik, Notodden, Norway*

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