DG 22: Current problems and challenges in university mathematics education

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Aims and focus

This report contains the conclusions of the deliberations of the members of DG 22. On the first day there were short plenary introductions to the themes of the DG. It was decided to concentrate on three themes, "widened recruitment," raising the profile of teaching", "dissemination of research." The participants from 18 countries were divided into three working subgroups, each charged with one theme. The task of each subgroup was to come up with a concise report on its deliberations, with concrete recommendations on how to improve undergraduate mathematics education with respect to its theme. Here is the resulting final report.

***Theme 1**. To address challenges originating from the fact that nowadays a greater proportion of the general population enters university and as a result, the background of students entering mathematics departments and those attending service courses in these departments has changed.*

There was agreement that in many of our countries elements of logic are not taught to the extent so that the senior high school students can tell a correct (or fallacious) proof when they see it or devise simple proofs of their own. In the United States, this problem is manifested when students move from computationally-oriented calculus courses to more theoretical, proof-oriented mathematics courses. In France it used to be the case that quite a bit of logic was taught at the secondary level because of the strong influence of Bourbaki. But the presentation was overly formal, and there was a backlash. Now very little logic is taught, and at the university level only a minimal amount about quantifier rules is given at the beginning of students' programs. The secondary school program in France has a big focus on geometry and students generally do well in this subject. But, nonetheless, when they arrive at university, they have difficulty recognising that when you have "If A then B" and you know B then you might or might not have A. In Algeria, students typically take calculus in secondary school but have a lot of trouble with the kind of quantified statements that occur in university coursework. For example, students have difficulty in proving that if $A \subseteq B$ then $f(A) \subseteq f(B)$, or that the limit of a composite of continuous f and g at a point a is f of the limit of g at a. Much of the problem is students' inability to work with quantified statements.

Recommendations. Students should be required to take a "transition-to-higher-mathematics" course. This is a course that introduces ideas of logical reasoning and proof but uses simpler topics than those in a theoretically-oriented calculus or analysis course. Another recommendation is to include proof techniques s part of a high school curriculum in order to improve students' logical abilities. A good geometry course prepares students to improve their logical reasoning abilities if careful use of definitions, theorems, and proof are emphasised. The role of semantics, in addition to syntax, needs to be emphasised at all levels.

What works to motivate students? The mathematical needs of non-mathematics-majors may vary significantly with the university and academic major. Too often in the past have we funnelled non-majors into some form of calculus course. In many instances this choice fits poorly the needs of our clients. We should endeavour to design service courses that actually fit the needs and values of our clients. Service courses should be regularly re-examined to ensure that they are kept timely and continue to meet the needs of the departments and students that they are intended to serve.

A variety of assessment practices can be used to motivate students. Project work, both individual project work and group project work, is used widely. Writing can be used not only to check how much students know but also to motivate their learning mathematics. For instance, writing an autobiography about their K-12 mathematics experience can be used to motivate students' learning. Well-designed forms of assessment can help students to see that mathematics is alive and active in many contemporary contexts.

***Theme 2**. To consider ways of raising the profile of the teaching component of an university career in order that it receive greater attention from researchers, universities and society at large.*

It is generally accepted that teaching excellence, in comparison to research achievements, generally plays little role in academic promotions. For example, the Dearing report in the UK in 1997 (downloaded from www.leeds.ac.uk/educol/ncihe/) asserts that the phenomenon of unbalanced rewards for research and for teaching was very serious for the quality of teaching throughout higher education. In the USA the faculty rewarding system is also a national issue (see www.aahe.org/initiatives/facultyroles.htm). Surveys in many countries have shown that in every category of staff and in every type of institution, there was widespread agreement that more emphasis should be placed on teaching than was currently the case. Next we cite some examples of practices that have been used with various levels of successes in various places.

Student participation in teaching evaluations. It is quite common practice for students to express their opinion of a class through questionnaires and written feedback. It is not clear, however, to what extent this practice is effective (e.g., is the questionnaire taken seriously? Does it differentiate between the students that like the teacher as a person and the students who think that he or she is a good teacher?) An interesting example of this type of assessment is the Student Evaluation of Educational Quality (SEEQ), of Curtin University of Technology. SEEQ is an instrument used to obtain student feedback on teaching and to develop teaching quality through reflective practice by the teacher. SEEQ recognises the complex and multidimensional nature of teaching and aims to provide feedback about teaching rather than content (see http://lsn.curtin.edu.au/seeq/index.html).

Helping young faculty in their teaching duties. An interesting example in the USA is Project NExT (New Experiences in Teaching) (see http://archives.math.utk.edu/projnext/). Project NExT is an MAA professional development program for new and recent Ph.D.'s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. In other countries initial teacher training programs for newly hired academic staff exist (for instance at most British universities as well as at Danish and Norwegian universities).

Team teaching. At Hanoi Technological University, at Greek universities, and other institutions, teams teach large classes and meet weekly to review class progress. In Gloucestershire University, teams of five teachers, from across different courses are formed, and these teams meet regularly to informally review each other's courses and teaching. In this way teaching issues and problems come under a broader departmental view and have an explicit focus.

Peer evaluation of research and teaching. At the University of Nebraska in the USA, every year each academic will sit down with the Head of Department and together they will agree on the proportion of time the person will spend on research, teaching and administration. Other faculty review each of these agreed upon aspects at the end of the year. The faculty member is rated from 1 to 5 on each aspect of work and this produces a score. Similar schemes are employed at the University of Maryland and Utrecht University.

Teaching awards and centres. For an excellent example of rewarding excellence in teaching see www.ncteam.ac.uk/ilts/publications/excellence.pdf. Another approach is the creation of centres for teaching development. Centres are places where faculty can find information, supporting material, consultancy etc. Such examples are the Derek Bok Centre for Teaching and Learning at Harvard (see http://bokcenter.fas.harvard.edu) and the Centre for Science Education at the University of Copenhagen (see www.naturdidak.ku.dk).

Theme 3. *To explore ways of disseminating the findings of research on undergraduate mathematics education to mathematicians – in particular, to promote learning from the theoretical advances in K-12 mathematics education research.*
This subgroup broadly explored the relationship between professionals in mathematics and professionals in mathematics education, with an eye toward examining ways to improve this relationship. For the purpose of this summary, Ms (mathematicians) refers to those professionals engaged in producing new results in mathematics and/or those who are primarily tertiary mathematics teachers, and MEs (mathematics education professionals) refers to those professionals in mathematics education producing research in tertiary mathematics education. Two small groups consisting of Ms and MEs from five different countries discussed three focus questions. The focus questions and the discussion of these are summarised below.

1. *What are the present relationships between professionals in mathematics and mathematics education professionals, and what made the present relationships as they are?*
The idyllic past: Mathematics as a discipline itself was less compartmentalised. Mathematicians from various specialisations somehow could understand each other. The atmosphere within the departments was more of the "liberal art" type, with small or no involvement from the outside. From the 1970's to the present, however, there has been more pressure on mathematicians to publish, more pressure to evaluate teaching, and move to mass university education. At the same time, MEs were professionalizing and specialising in primary, secondary, and more recently tertiary education. This move to specialisation may have inadvertently fostered a splintering between Ms and MEs.

Ms recognise the need to improve collegiate mathematics education. In that respect, they recognise mathematics education as an important and useful (if not necessarily scientific) discipline. Many Ms, however, do not accept MEs, while on the other hand Ms goals and beliefs are often not sufficiently considered by MEs. A common critique of reform-oriented K-12 mathematics programs is that teachers' beliefs, goals, and experience are de-valued or ignored. If Ms beliefs and goals are at variance with MEs beliefs and goals, then there will be difficulty in their interpretations of MEs results. In particular, MEs tend to focus on process (e.g., how a point is argued) rather than on content (e.g., what algorithms/concepts are taught). If Ms cannot appreciate this distinction, they will fail to see the point, value, and legitimacy of ME work.

2. *Why don't some professionals in mathematics consider research in mathematics education to be "scientific" or of value? To what extent are these and related views justified?*
Some Ms feel MEs lack the credentials to do the work that they do. How can someone who has not taught real analysis tell an Ms how to teach real analysis? How can an ME tell someone who proves for a living about the epistemological nature of proof? Mathematics Education, along with other social sciences, is less precise and less objective than Mathematics and Sciences. If you insist on defining, as many Ms would tend to do, "scientific" by being rigorous, systematic, and having exacting standards, then mathematics education is less scientific than Mathematics and Sciences. Ms are by training, and perhaps inclination, going to be more comfortable with quantitative research. Today MEs work is primarily qualitative. As a consequence, when MEs results are described in a way so that they are accessible to Ms, they are perceived to be common sense.

Another way to address this question is as follows: Even mathematics disciplines must go through a lengthy period before they are accepted by the mathematical community. For instance, Cantorian set theory and, more recently, category theory. It is only natural that mathematics education should have to go through such a period before being accepted as a legitimate discipline by Ms, especially since it is so different from other mathematical disciplines.

3. *How can the relationship between professionals in mathematics and professionals in mathematics education be improved?*
One suggestion is to try to improve the relationship at a local rather than a global level. It is natural in any discipline that journal articles are tough to read (they are densely packed, employ jargon, and are trying to move a field forward) while face-to-face interaction moves more smoothly. The group envision changes in Ms views of ME coming department by department, or mathematician by mathematician. In short, the group want to change the interactional paradigm between mathematicians and mathematics educators. Five ways of doing this were proposed:

1) Have mathematics educators as faculty members of mathematics departments, perhaps through joint appointments.
2) Have MEs "revamp" courses that they have had experience teaching with input from mathematicians.
3) Offer colloquia discussing teaching between Ms and MEs.
4) Hold seminars in which Ms and MEs each read articles on mathematics education and then discuss them.

5) Develop reports of qualitative research accompanied by high quality videotape, when available. Videotape more sharply illustrates the value that innovative mathematics education courses can do in a way that transcripts cannot.

At the global level, it is useful to work more broadly by establishing pedagogical centres/didactical units in which Ms and MEs can interact and work on problems of teaching and learning together, with each bringing their own expertise.

Ms must see a need for MEs. This need must be both internal and external. Internally, it is critical for Ms to perceive something as problematic in their teaching practice. Externally (from administrators and from the public at large) Ms are beginning to receive pressure to seriously consider aspects of their teaching, to make improvements to their teaching, and to demonstrate impact on student learning and attitudes toward mathematics. Both internal and external pressures can promote a mutual need between Ms and MEs.

The products from MEs (both theoretical and practical products) must be of real use to Ms. Products from MEs should be adaptive, rather than prescriptive. Three additional suggestions for continued improvement are (1) to develop publication(s) of MEs products specifically for Ms. Ms are not interested nor do Ms have the background to read original research reports written for other MEs; (2) to develop collaborations between MEs and Ms centred around problems and challenges of tertiary education; and (3) to explore possibilities of adapting a "lesson study" type collaboration between Ms in collaboration with MEs.

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