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Thematic
Afternoon D

TA D: Technology in mathematics education

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Introduction: Overview of the theme

The thematic afternoon “Technology in mathematics education” provided an opportunity for participants to find out about current and future technologies, to focus on how ICT can be and is being used for teaching and to engage with current research perspectives from around the world. Novice users of technology in education were able to use the afternoon as an opportunity to see the potential for ICT to enrich mathematics teaching while experts could further explore current and emerging issues. There were sessions of relevance to all levels of education: elementary, secondary and tertiary.

Through a range of lectures, panels and hands-on sessions, the afternoon addressed four major topics for elementary, secondary, tertiary and teacher education:

- New developments in information and communication technology for mathematics education
- Advantages and pitfalls concerning technology in mathematics education
- The Internet and mathematics education: Accessibility, use and misuse
- Technology in distance teaching and learning.

In keeping with the wide-ranging nature of the theme, there were a total of 17 sessions and 52 speakers, addressing highly diverse topics from many different perspectives. Six hands-on workshops for groups of about 30 participants enabled participants to obtain direct experience of new software products and new uses of established technologies for all levels of schooling, including teacher education. Simultaneously, there were 11 lecture hall sessions, mostly lasting 105 minutes, where speakers demonstrated new possibilities and discussed some of the many pedagogical questions that arise when technology with mathematical power is put into the hands of students. Both information and communications aspects of ICT were featured, since both these areas have seen significant development since the last ICME. Recent developments include a widening of who has access to ICT (although this remains a source of great inequity between and within countries), a greater sophistication and range of mathematical tools and imaginative teaching devices and the growth of internet resources and communication. The descriptions below indicate in broad terms the major concerns and interests of the sessions.

Introducing ICT: Experiences and issues

This session, chaired by *Kaye Stacey*, drew together experiences from around the world on productive ways to introduce technology in school systems. From practical and theoretical viewpoints, it addressed issues such as equity and teacher training and the nature of systemic support needed to make the introduction of ICT positive for school systems. Sharing experiences from many countries, and appreciating the differences, was a major outcome of the session. There are common problems but they are experienced



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in different ways and to differing extents. *Wong Khoon Yoong* (Singapore) described Singapore as an example of a technologically rich learning environment. Schools are well equipped, official support to use ICT is strong and there is adequate in-service training. Wong attributed the lower-than-expected uptake of technology use in classrooms to the pressure to prepare pupils for public examinations.

Some countries have addressed this pressure by exploring how the public examination system may be changed so that the assessment itself promotes technology use. *Peter Flynn* (Australia) provided an example of this. From 2002, student use of CAS calculators has been permitted in some high-stakes assessment. Assessment cannot be left unchanged when the learning environment is different, but the changes require mathematical, pedagogical and ethical considerations.

There were three presentations from countries where the economic circumstances severely restrict access to technology. *Luckson Kaino* (Botswana) reported on ICT availability and utilization in Botswana primary and secondary schools, with insights for other developing countries. *Yuriko Yamamoto* (Brazil) discussed introducing hand-held technology in mathematics classrooms of basic schools in Brazil, a developing country with many economic and social problems. The main challenges are to guide teachers to discover the didactical potential of technology and to link previous content knowledge with technology-appropriate activities. *Louise De Las Peñas* (Philippines) explained how, since the late 1990s, technology has slowly been introduced to mathematics teaching in universities and schools and nationwide teacher training and outreach programs have been conducted. She identified the main challenges confronting teachers in the Philippines as equipment, learning environment and curriculum.

Whilst the above presentations served to highlight inequities between countries, *Penelope Dunham* (USA) reminded us that inequities are also evident within systems. Inequities arising from differential access to and use of educational technology for groups characterized by gender, race/ethnicity, or social/economic class can limit the impact of ICT. She suggested public policies and pedagogies that may remove the boundaries between technology “haves” and “have-nots”.

Showcase surveys

Three lecture hall sessions and most of the workshops showcased exciting new possibilities for teaching and learning with ICT. *Robyn Pierce* (Australia) chaired the session entitled “Advances in undergraduate education with ICT”. *Neil Challis* (UK) gave examples to illustrate that we must encompass symbolic, graphic and numeric thinking, and he emphasised that “doing mathematics” includes the whole problem solving process from its initial source to solution and appropriate communication of conclusions. Robyn Pierce gave examples from her teaching of both mathematics and statistics to demonstrate how technology can support and enhance the learning of diverse student cohorts, by promoting better understanding of concepts and providing access to real world problems. *Jack Bookman* (USA) demonstrated how students negotiate roles and meaning as they learn in technologically rich environments and compared active learning in technologically-rich and pencil and paper environments. *Paul Igodt* (Belgium) demonstrated a web-platform and database architecture for multiple choice problems and course-specific tests which allows sharing of exercises between teachers and courses. By using so-called “learning objects meta-data”, the high effort requested from authors of exercises gets a beneficial return in ease of sharing and ease of reusing over time.



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The session “Teaching primary and junior secondary mathematics with ICT: Changing pedagogy and learning”, chaired by *Kaye Stacey* (Australia) also showcased mathematics and statistics examples. *Kaye Stacey* presented a theoretical overview of how technology in classrooms can operate to increase engagement and achievement of students, in particular by bringing real world problems to life in the classroom, and she gave examples of doing this with digital and video images. *Douglas Butler* (UK) showed how a teacher can use one computer in a classroom to enliven demonstrations and problem solving. Use of dynamic images can play a crucial role in inspiring pupils to want to be more successful, and therefore to want to take the subject on at a higher level. *Tim Erikson* (USA) used statistics software to capture and analyze data from the internet. By choosing the data and the context carefully, we help students make use of the mathematics they are learning already, to accomplish something of genuine interest to them. *Jenni Way* (Australia) demonstrated a new suite of digital learning objects for the first years of school, to be delivered to every school in Australia and New Zealand on demand.

Ricardo Nemirovsky (USA) chaired a session that showcased learning mathematics with physical phenomena and involving kinaesthetic, bodily experiences. *Ornella Robutti* (Italy) reported on teaching experiments logging body motion with sensors and calculators from kindergarten to secondary school. *Karen Marrongelle* (USA) reported on investigations of the interface between students’ understandings of the integral and subsequent performance on physics problems. The use of the integral in physics is not simply the application of a mathematical technique to numerically solve a problem but needs to be reinterpreted for each problem situation. *Michal Yerushalmy* and *Beba Shternberg* (Israel) showed software to develop the concept of function from physical experiences with technology. *Apolinario Barros* (USA) reported how kinaesthetic activities with a 2D motion detector can assist students to understand the relationship between sine and cosine.

Two workshops showcased classroom activities using dynamic geometry packages. *Sophie Soury-Lavergne* (France) explored new types of tasks made possible by dynamic geometry, emphasising the new ways in which mathematical properties appear to the learner when the new tool is used. *Nicholas Jackiew* (USA) extended the use of dynamic geometry from investigations of shape to number patterns and elementary number theory, fractions and early algebraic reasoning.

The algebra theme

In recent years, algebra teaching has been significantly influenced by technology, so four lecture hall sessions and three workshops specifically addressed this. The influence is due to the way in which software gives students access to symbolic, numerical and graphical representations of algebraic ideas. It is also due to the fact that teaching and curriculum has to adjust to a technological environment where many of the calculation aspects of algebra can be taken over by technology, although this needs to be handled carefully to get the best for learning.

Jean-Baptiste Lagrange (France) chaired a double session, which reviewed a range of technologies that can enhance understanding of algebraic ideas and track students’ progress. *Carolyn Kieran* (Canada) described research on algebra learning and teaching that has been carried out in various technological environments, with either multiple representations, dynamic control, or structured symbolic calculation. The duality of



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algebra with its multi-representational functional approaches on the one hand, and symbol-based manipulation perspectives on the other, provided a framework. *John Olive* (USA) showed how to connect motion, geometry and algebra using dynamic geometry and simulation software. *Jean-Baptiste Lagrange* (France) considered the place of classroom situations involving experimental approaches and algebraic treatment of functions with the help of technology. Although the use of CAS seemed promising, difficulties were experienced and curricula now tend to privilege approaches to functions with non-symbolic software. *Brigitte Grugeon* and *Elisabeth Delozanne* (France) presented prototypes of software for diagnosing students' competencies and building cognitive profiles for algebra, which is based on an artificial intelligence approach. *Rosamund Sutherland* (UK) focussed on using spreadsheets for enhancing the learning of algebra. She identified a gap between students' idiosyncratic approaches to solving problems and a more socially accepted school algebra approach, and explained why the use of ICT could exacerbate this problem. *Alain Bronner* (France) reviewed various uses of the "Aplusix" software for learning to solve equations or systems of equations. He analysed the role of the various interactions with the technological environment in the evolution of the concept of equation and strategic knowledge of the students to solve equations.

"The teacher and the tool", chaired by *Paul Drijvers* (The Netherlands), presented findings from three research studies which have examined how teachers adapt to using CAS in their classrooms. *Lynda Ball* (Australia) asked what "assessment" looks like in a CAS classroom. She explored the ability of students to communicate solutions when intermediate steps were assigned to CAS and how to help students communicate well. *Bärbel Barzel* (Germany) noted that the integration of CAS in teaching mathematics leads quite often to a change from classical instruction to a more constructivist approach. Both of these changes, integrating CAS and opening up learning, can be challenging for teachers. *Rose Mary Zbiek* (USA) offered several insights into the complexity and promise of teaching with CAS. Examples of classroom-teaching events lead quickly to factors that impinge on teaching with the technology. She described how the multi-faceted relationship between and beyond the teacher and tool emerges from mere acquaintance to effective partnership.

In a second session, entitled "Instrumentation and CAS" two views were presented on the process by which teachers and students come to be skilled users of computer algebra technology. *Paul Drijvers* (The Netherlands) began by distinguishing an artifact from an instrument and describing the process of instrumental genesis by which the transition is made. This instrumental genesis includes the development of mental schemes for using the artifact for the target activity. In such schemes, technical and conceptual aspects interact. Then *Luc Trouche* (France) provided evidence to show that the more complex the environment, the more diverse the students' work methods, and, consequently, the more necessary the 'orchestration' of the teacher in order to assist instrumental genesis.

Two workshops gave participants first-hand experience of integrated mathematical environments with substantial mathematical power. *Renée Gossez* (Belgium) gave an introduction to the power of CAS in teaching for school years 9 and above. She examined use of capacities such as the automatic updating of documents, the use of sliders to change the values of parameters, the capacity to pass worksheets between the students and teacher, and the substantial mathematical calculation capacities. *Steen Grode* (Denmark) demonstrated teaching mathematics with "Mathcad" and "Smartsketch".

Together these make an integrated environment for performing and communicating math-related work, which has been trialled in Danish schools.

Internet resources for teaching mathematics

Although use of the internet featured in many sessions, *Shoichiro Machida* (Japan) chaired a session which specifically presented internet resources from around the world for students at all levels of education and their teachers. *David A. Thomas* (USA) observed that modelling technologies are empowering students to formulate, test, and support their own mathematical conjectures. Integrated mathematical modelling and web-based communication technologies can help achieve better mathematical dialogue. *Akimichi Tanaka* (Japan) demonstrated a tool called “linkWorks,” which helps learners search for information related to given subjects on the Internet and collaborate with each other. Students using the tool learned actively and collaboration among them raised their learning motivation. *Vincent Jonker* and *Frans van Galen* (The Netherlands) demonstrated the “RekenWeb”, a website providing mathematics internet games for primary education and printable activity sheets for teachers, which provides many opportunities to support teachers for their daily lessons. *David A. Smith* (USA) described the Journal of Online Mathematics and its Applications (JOMA). JOMA contains articles, modular learning materials, reviews, “mathlets”, and a Developers’ Area for assistance in creating online materials.

One workshop showcased small stand-alone software programs called applets, to use across the internet. Drawing their examples from teaching introductory algebra, *Peter Boon* and *Martin van Reeuwijk* (The Netherlands) illustrated model applets for concept development and practice applets to reinforce skills.

Distance learning

Shoichiro Machida (Japan) also chaired a session on developments in distance learning; an area which has undergone rapid change in recent years. Machida reported on a digital learning environment for supporting teachers to encourage students’ self-directed learning in the mathematical classroom. Hypermedia mathematical textbooks, called e-subtextbooks, included a section that is regenerated by teachers themselves every lesson. Teachers’ group collaboration was supported through a mailing list. *Shuhua An* (USA) reported on teaching mathematics methods for pre-service teachers by an integrated hybrid course, combining the best of traditional and on-line teaching. It was particularly appropriate for independent, focused, and goal-oriented students. *Lyn Leventhall* (UK) reviewed collaborative teaching resources on the web and “web ready” software using the underlying technology “MathML” for displaying mathematical equations. *Masami Isoda* (Japan) reported on a different style of distance collaboration, which involved students from different countries collaborating to solve mathematical problems via regular e-mail exchange. The projects illustrated that the major significance of communication between countries is cultural awareness in mathematics. Mathematics is a communication tool and developing communication ability is an important aim in mathematics education.

Video-based technologies in teacher education

In recent years, there have been significant changes in the ease of creating, storing and accessing video-based information. Video is now a highly practical, as well as extremely



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rich, data collection tool. This has had a major impact on teacher professional development, pre-service education and also on research. *Ricardo Nemirovsky* (USA) designed and chaired the session. *David Clarke* (Australia) discussed the use of video material for mathematics teacher education in a climate of standards-based reform. Professional standards have the potential to communicate the findings of research and the wisdom of practice in the most practical manner, but also have the potential to become prescriptive and constraining. *Chronis Kynigos* (Greece) illustrated how teachers' epistemology and perceptions of teaching and learning mathematics were challenged through their interactions with exploratory software during a professional development course. He believes that it is important to perceive technology as a medium for the empowerment of teachers, rather than just a powerful tool for students. *Robert Tinker* (USA) reviewed the provision of online teacher professional development, noting that it has great potential but has been marred by poorly designed and executed courses. He described an on-line course for algebra teachers using video case studies and software tools delivered during the school year, and presented evaluation results.

The workshop with a focus on teacher education, offered by *Federica Olivero* (UK) and *Dan Cogan-Drew* (USA) described how self-study projects using the "VideoPaper Builder" software can be used to teach mathematics pre-service teachers to reflect on their practice. This is directed use of video for self-study, not for use by others. The workshop discussed progress on transforming videopaper creation into a new teacher education model.

Conclusion

Overall, the field is characterised by rapid change, as new products and possibilities become accessible to more people. Educational responses are strong, both to mould new opportunities to improve learning at all levels, including teacher learning, and to refine pedagogical practices to strengthen the value of technology in teaching.

This report has been written by Kaye Stacey and Paul Drijvers. They will be happy to be contacted at the University of Melbourne k.stacey@unimelb.edu.au and The Freudenthal Institute p.drijvers@fi.uu.nl for further information on the work of this Thematic Afternoon.