TA E: Perspectives on research in mathematics education from other disciplines

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Introduction

The thematic afternoon on perspectives on research in mathematics education from other disciplines provided an opportunity to focus on the contributions of psychology, cognitive science, philosophy, sociology, anthropology and general education to research in mathematics education. Such contributions include theories, issues, problems, concepts, methodologies, studies, and results that are of significance to the international mathematics education research community. An overview of such contributions was complemented by accounts of specific research projects incorporating such elements from outside disciplines. These external references balance the traditional attention to 'homegrown' (versus 'imported') theories (and by extension, concepts, etc) in mathematics education research. Here we were explicitly focussing on the role of such intellectual imports and appropriations, and illustrating them with exemplary mathematics education research projects, as well considering the overall role of external disciplines in our work.

In addition to the Team Chairs *Brent Davis* and *Paul Ernest* there were three further plenary speakers: *Tommy Dreyfus*, Tel Aviv University, Israel; *Christine Keitel-Kreidt*, Free University of Berlin, Germany; *Robyn Zevenbergen*, Griffith University, Australia.

Summary of Strand 1: The perspectives of psychology and cognitive science in research in mathematics education

"Psychology and cognitive science" sweep across neuroscience, cellular biology, developmental psychology, linguistics, and cultural anthropology – to name only the few disciplines that were explicitly invoked in Strand 1 presentations during the Thematic Afternoon.

The fact that these topics should be included among so many others during an afternoon devoted to the exploration of other domains highlights how things have changed within mathematics education research over recent decades. There was a time that this field looked almost like a subset of psychological research. In fact, the learning of mathematics is still a favourite phenomenon of study among cognitive psychologists. But psychology is no longer such a favourite domain of inquiry among mathematics education researchers.

Each of the contributors to Strand 1 looked at a different and quite distinct phenomenon, as one might expect given the very different discourses that frame their work. That said, however, despite the clear differences in objects of interest, there were some striking and provocative similarities in the manners of description offered. By way of a conceptual organizer, and as became very apparent through the course of the presentations, the phenomena of interest in Strand 1 seem to be nested in one another, beginning

with subpersonal phenomena, and moving through personal, interpersonal, and transpersonal. So framed, and despite at least one major tension that arose, it was clear that there are deep complementarities among the discourses invoked. They need not be treated as competing, or even disparate fields, but as overlapping and intertwining areas of inquiry that might better be considered in terms of their collective contribution to mathematics education research than in terms of their particular foci.

For example, the first presenter, *Daniel Ansari* of Dartmouth College (United States), started the strand presentations with a review of current research in cognitive neuroscience into children's development of numerical and mathematical skills. Daniel argued that early developing approximate number skills contribute to the gradual development of exact number representations and that these systems are represented differentially in the adult brain. Among the consequences of this research, Daniel argued that the findings urge educators to place greater emphasis on early education of basic numerical skills and how they point to the importance of basic quantity understanding.

Daniel's topic, while focused on the subpersonal, clearly pressed into the space of personal understanding, which is where *Willy Mwakapenda* of the University of Witwatersrand (South Africa) located his presentation. Willy focused on concept mapping, which he offered as a methodology for researching student understanding. Through a series of examples, Willy argued that students' understanding of concepts is highly related to the contexts and experiences in which they learnt mathematics – a conclusion that pressed the discussion into the space of the interpersonal.

That was the principal site of the third presentation, from *Joyce Mgombelo* of Brock University (Canada). Joyce argued for the significance of Lacanian psychoanalysis for mathematics education research, contending that Lacan's (1977) distinction of objective knowledge and knowledge-as-enjoyment presents a significant reframing of questions of knowing, knowledge, and experience. Focusing her interpretations with the 19th-century writings of Mary Boole, Joyce looked at the relationship of the mathematics teacher and her or his students.

Thomas E. Kieren, University of Alberta (Canada), moved us from the space of the interpersonal to the transpersonal. Specifically, Tom's contribution was concerned about "conversations" among fields of inquiry. He argued that the influence of one domain on another can never by unidirectional – that is, that the mathematics educator not only takes on, but necessarily transforms ideas developed elsewhere, which in turn presents the potential for the changing of the ideas from the home field as well.

Part of the character of such interdisciplinary conversations was powerfully illustrated in the final presentation of Strand 1. *Terezinha Nuñes* of Oxford Brookes University (United Kingdom) drew on developmental psychology to critique the use of any single discourse, in particular cognitive neuroscience, to make sense of human competencies that arise in and unfold through the interweaving of biological processes and the invisible symbolic web of culture. Such competencies, she argued forcefully, must be studied in their wholeness, neither as reducible to subpersonal processes nor as by products of more global processes.

Summary of Strand 2:
The perspective of philosophy in research in mathematics education

What might the perspective of philosophy in research in mathematics education mean? Philosophy involves a critical examination of fundamental problems and assumptions, and systematic analysis, reasoning, judgement, resulting in conclusions, knowledge and beliefs. However, these are what we expect of good research in mathematics education too, although philosophical enquiry may be more thorough, or may pay more attention to the process of conceptual clarification itself. The unique contribution of philosophy is the use of the substantive concepts, theories and results of past philosophical enquiry.

Research problems in mathematics education are typically multi-faceted and require an awareness of the complexity of the teaching and learning of mathematics and the surrounding social context. So philosophy cannot usually be applied directly to solve such problems. *Anna Sierpinska*'s (Concordia University, Canada) contribution "The philosophical perspective in mathematics education" pointed out the danger that philosophy may end up leading to generalities, and understanding nothing in particular rather than understanding specific mathematical concepts.

Where most successful in applying philosophy, researchers in mathematics education draw upon philosophical theories and concepts as resources to help clarify research problems and their conceptual frameworks. Typically they start with a problem in mathematics education research, then search for resources drawing on relevant philosophical concepts and theories before importing and adapting them in constructing and clarifying a conceptual framework for their research. The following are brief sketches of sample uses of philosophy in maths education

Philosophy of mathematics. It is well known that there are different (and contrasting) philosophies of mathematics. 'Postmodern' developments in the philosophy of mathematics have been concerned with mathematical practice and what mathematicians do to create (and justify) new mathematical knowledge (e.g., Lakatos, Davis & Hersh, Tymoczko)

These developments have been a useful resource for mathematics education researchers wanting

1. To give a dynamic and humanistic account of mathematics
2. To find philosophy of mathematics compatible with problem solving in the classroom
3. To research the processes of doing maths. *Leone Burton*'s (University of Birmingham, United Kingdom) contribution in this area was "Mind the gap" and was about her work in exploring knowledge and knowing, epistemology and pedagogy, in mathematicians' practices.
4. To find a philosophy compatible with multiculturalism and ethnomathematics. *Bill Barton*'s (University of Auckland, New Zealand) contribution in this area was "Culture and mathematics" and explored philosophical perspectives concerning anthropology and ethnomathematics and mathematics education.

Personal knowledge and knowing. Researchers have been concerned with what it means to know mathematics and different forms of knowledge. One important distinction due

to Gilbert Ryle is between 'knowing that' and 'knowing how'. A number of philosophers and researchers in mathematics education have made the distinction between explicit vs. tacit forms of knowledge, including: Polanyi, Kuhn, Wittgenstein, Skemp, Mellin-Olsen, Hiebert *et al.*, Kitcher, and Ernest. Several of these, especially the last two, have proposed multi-dimensional models of mathematical knowledge encompassing such distinctions. *Cristina Frade* (Universidade Federal de Minas Gerais, Brazil) drew upon several of these latter authors in reporting her investigations of the tacit-explicit dimension of the learning of mathematics.

Research methodology and paradigms. The philosophy of science, especially the work of Popper and Kuhn, has been influential in educational research, especially in the scientific research paradigm. *Gerald Goldin* (Rutgers University, USA) made his contribution entitled "Toward reproducibility and generalisability" in this area, where he offered perspectives on mathematics education research from the philosophy of science.

In addition to this important area of philosophical influence, thinkers including Weber, Schutz and Habermas have contributed much to the philosophy of the social sciences underpinning the interpretative research paradigm, sometimes called the qualitative paradigm. Habermas and the Frankfurt school have also led to the foundation of the Critical-Theoretic research paradigm in education research. Thus philosophy has been especially important in the area of educational research methodologies.

Although these areas included all the individual contributions to the philosophical strand *Paul Ernest* also indicated a number of other important areas of influence and controversy. These included the following.

Theories of learning. There are different philosophical traditions underlying theories of learning. Empiricist theories stem from Locke, Hume and Mill. Constructivist theories can be traced back to Kant and Piaget. Social theories are more recent, and can be found in Mead, Wittgenstein, and Vygotsky. Heated controversies over theories of learning mathematics still abound, with empiricism, cognitivism, radical constructivism, enactivism and embodied cognition, social constructivism and socio-cultural theories of learning still slugging it out. As Ernst von Glasersfeld (1983) said, to introduce epistemological considerations into a discussion of education has always been dynamite.

Ethics, values, feminist theory. Research on gender and mathematics education has been strongly influenced by philosophical theories of moral and epistemological development. The strongest inputs have been from Gilligan (1982), and Belenky *et al.* (1986) distinguishing between separated and connected values, separated and connected knowing. However, not all of the potential for growth in mathematics education research based on feminist theories has been realized yet, in *Paul Ernest*'s opinion.

Several other areas of philosophy have the potential to further influence and contribute to research in mathematics education. For example:

1. Philosophy of biology – this is important for enactivism and embodied learning.
2. Postmodernism, post-structuralism and political theory – these can contribute much on researching power and the social impact / context of mathematics education.

3. Philosophy of language – the interpretation of discourse is an increasingly important dimension of mathematics education research.
4. Hermeneutics – has likewise much to offer on textual interpretation.
5. Semiotics – can further contribute to a deeper understanding of the sign systems of mathematics and mathematics education.

Summary of Strand 3:
The perspective of the social in research in mathematics education (incorporating sociology, anthropology and general education)

The contributions introduced by *Robyn Zevenbergen* were the following.

1. *Tine Wedege* (Malmö University, Sweden): "Import and reconstruction of concepts: the social dimension of mathematical knowledge"
2. *David Wagner* (University of Alberta, Canada): "New directions for analyzing mathematics classroom discourse"
3. *Paula Ensor* (University of Cape Town, Republic of South Africa): "Sociological perspectives on research and practice in mathematics education"
4. *Derek Woodrow* and *Janis Jarvis* (Manchester Metropolitan University, UK): "Learning preferences of mathematics students compared to students of other subjects"
5. *Ubiratan D'Ambrosio* (Unicamp, São Paulo, Brazil): "Is integrating science and mathematics a promising option?"

In the final hour the three strands were brought together in a shared plenary session chaired by *Christine Keitel-Kreidt*. In addition to brief comments from the three strands, a presentation was made by *Tommy Dreyfus* addressing "The power of homegrown theories in the discipline of mathematics education." His presentation serves as an important cautionary tale that encourages us to look elsewhere for ideas when researching in mathematics education, but reminds us not to be overly committed to or swayed by any singular domain, or to lose sight of the complexity of the phenomenon at hand.

It was a rewarding if densely packed afternoon. Naturally there are problems associated with an afternoon conference-in-a-conference that is about every other field of inquiry except the one that serves as the focus for the rest of the conference. In addition there are many other fields that might – and, in fact, do – inform work in mathematics education research, and that were, by necessity, ignored here. Every field that we did manage to address has a terrain that is as varied and as contested as that of mathematics education research.

One of the things that came through pretty powerfully was that we need to be mindful of the discourses that we draw on, and in particular, the subpersonal, personal, interpersonal, and transpersonal consequences of what we bring together.

One final concluding thought concerns where discourses are focused. Educational research has to be educational – it has pragmatic concerns that other domains of inquiry do not have. For that reason, drawing from domains in which discussions can be described to be mainly descriptive in character, rather than pragmatic, means that we are borrowing ideas that are not educational – that is, they are not framed, by the educators' pragmatic concerns. And if we are not careful with that, history shows that problems will arise.

References

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