TSG 3: New developments and trends in mathematics education at tertiary levels

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Introduction

Many of today's undergraduates cannot imagine life without cellular phones and laptops. How do these rapid technological changes in our society influence teaching and learning mathematics at the tertiary level? While many of mathematics lecture halls are still dominated by instructors "chalk and talk" and students' hasty note-taking, others engage in creative explorations, the use of technology and problem solving. But are there any identifiable developments and trends, either local or universal?

The aim of our Topic Study Group was to explore this issue of recent trends and developments from around the world. We adopted the format of presentations and discussions in an effort to aid this exploration. We attempted to balance our desire to provide a comprehensive overview of the state-of-the-art with an understanding that the time frame of the Group only allowed for "snapshots". With a considerable effort from all the team members we managed to assure the geographical variety of these snapshots. The presented papers illustrated work from Canada, Germany, Hong Kong, Israel, Japan, New Zealand, Russia, South Africa, the United States, and Uruguay. (See below for a detailed list of papers presented or distributed for TSG 3.) The work of the group laid a foundation to a special double issue of the International Journal of Mathematics Education in Science and Technology that appeared in 2005.

The topic of tertiary mathematics education is extremely broad not only because of our geographical variety, but also because of variety in content. "Tertiary" is more of a place – colleges and universities – rather than "level". Learning mathematics at tertiary level there are future research mathematicians taking advanced abstract algebra courses, future users of mathematics struggling with business calculus, liberal arts students relearning basic algebra to comply with "numeracy" requirements, and future teachers of mathematics, to mention just a few target groups and levels. In what way are the recent trends and developments similar or different for these groups of learners?

Selected points from the presentations

The work of our group at ICME 10 started with the presentation by *Annie Selden*, who put forward a provocative title for her paper: "New Developments and Trends? Or, More of the Same?" Selden's presentation served in *setting the stage* for the Topic Study Group. She identified four major topics of interest in undergraduate mathematics education. These were: the role of technology; the transition from secondary to tertiary education; the need to produce future mathematics teachers: and the potential impact of research into teaching and learning at tertiary level. She then pointed to a subset of these topics that was addressed at the congress and foreshadowed many of the forthcoming presentations, situating them in a broader context of tertiary mathematics education. In an attempt to cluster the work of the group in some reasonable way we identified several overlapping themes related to the theory and practice of mathematics education: issues of transition as related to both curriculum and pedagogy, teacher education, research in undergraduate mathematics education, technology and its influence on curriculum, pedagogy and research. We acknowledge that consideration of these themes overlaps with other working groups at ICME, in particular with those on teacher education and technology. However, considering the issues of concern to tertiary mathematics education, we find this overlap unavoidable.

The issue of challenges in *transition to undergraduate and advanced mathematics* was a significant focus in the work of the group. Lovric pointed to a trend that school graduates are less prepared in dealing with university level mathematics. This observation was based on comparing two cohorts of students: those with 5 years of secondary school education and those with 4 years, where the latter is the result of the recent imposed change in the province of Ontario, Canada. A similar argument was echoed by *Nishimori* based on the results of a survey among university faculty in Japan. This survey suggested deterioration in the problem solving skills of students as well as in their algorithmic fluency. Luk provided a personal account of transition, describing the challenges he faced both as a student and as teacher of undergraduate mathematics in Hong Kong. *Hockman* presented a concern of "watering down" courses in order to comply with the need to accommodate a larger amount of students and the lack of support from administration in South Africa. These papers raised a universal concern – the concern of deterioration – that was mirrored by several comments and reflections on the personal experience of participants.

Responding to the question presented by Selden – "New Developments and Trends? Or, More of the Same?" – we admit that the issue of the transition to undergraduate mathematics is not "new" to the field, but it has been reinforced and aggravated, and has received a new attention recently, considering the growing amount of students entering tertiary education and diminishing amounts of funds to support these students. Selden referred to the problem as "two contradictory trends": on one hand there is advocacy from the academic community for school graduates who are better prepared mathematically for both university and the work place, on the other hand there is evidence that legislatures and administrative bodies around the world are reducing requirements for both high school diplomas and university degrees.

There is an ongoing effort to develop curriculum and pedagogy to address better the *needs of all students* as well as of specific populations of students. These efforts were featured in the papers by *Paramonova*, who outlined the curriculum at the Moscow Independent University that serves to educate future research mathematicians, and *Safuanov*, who presented a view on pedagogical development implementing a "genetic" approach to teaching, that is, pedagogy that recognizes historical, logical and socio-development of the subject matter. Further, acknowledging that "understanding" is the ultimate goal of teaching mathematics, Kannemeyer provided an innovative instrument attempting to identify what such understanding entails in the context of a calculus course. The paper by *D'Arcy* presented strategies that may help students in memorizing the mathematical contents they are studying.

Teacher education was another important focus in our Group. After all, considering tertiary mathematics education, a significant part of it is education of future teachers of mathematics. The work of the group related to teacher education considered issues in curriculum, pedagogy and research. *Martinez Luaces* presented a case for the use of modelling in the curriculum for mathematics teachers and provided several examples of modelling activities that seemed to have a positive impact on the participants who engaged in these activities. *Wittmann* presented what he referred to as a "notion of operative proof", that is, a proof that introduces the ideas behind the mathematical argument without relying heavily on formalism and symbolism. He made an argument for using operative proofs as part of the pedagogical approach with pre-service teachers. The papers of *Zazkis* and *Liljedahl* presented reports on research conducted with the population of pre-service teachers. Zazkis investigated the ways in which students perceive irrational and prime numbers and pointed to common features of these two sets considering how these numbers can or cannot be represented in a standard algebraic notation. Liljedahl investigated the impact of successful mathematical discovery – referred to as an Aha! experience – on the beliefs and attitudes of pre-service elementary teachers. *Leikin*'s research considered both undergraduate and graduate mathematics education students and acknowledged the similarities in the interaction between a teacher and a student and between a mentor (supervisor of student-teachers) and a mentee. Leikin presented an example of connection between theory and practice of mathematics education, specifically, how a model of interaction developed by analyzing the work of in-service teachers can be used with pre-service teachers in order to raise the quality of their discourse about teaching.

Returning once again to Selden's question we believe that some novelty can be claimed here on two accounts. First is the relatively novel and growing attention to *research* in undergraduate mathematics education. Another is the attention to the *teacher*. The latter is in accord with the claim made by Anna Sfard in her plenary presentation, "There is nothing more practical than good research: On the mutual relation between research and practice in mathematics education", that identified the current decade in mathematics education research as "the decade of the teacher", while the previous two decades could be considered as the "decade of the curriculum" and "decade of the learner".

Technology is the theme that intertwines with all the areas on mathematics education. As a snapshot of the influence of technology on curriculum and pedagogy Hillel considered the case of Linear Algebra, he described in what way various computer based activities and assignments are used to further students' understanding and appreciation of the subject. Web-based or web-supported courses are a paradigmatic example of how technology influences the pedagogy of course delivery. *Engelbrecht* and *Harding* presented a classification of courses that rely on the internet in various ways – ranging from reference to the web for illustrative examples in a "standard" mode of delivery to a full course delivered electronically – and discussed the impact these courses have and may have on undergraduate education. As a snapshot from the possibilities offered by the world wide web, a paper by *Zgraggen* presented a programme that provides students with dynamic guidance to solving problems. As a snapshot from research that investigates the influence of technology on learning, *Gurevich*, *Gorev* and *Barabash* studied the impact of the use of various computerized tools on students' achievement in plane geometry and in analytic geometry.

So, again, is it "More of the same"? The "sameness" is in the idea that technology is one of the forces that is driving the change in curriculum, in pedagogy and in research. This is hardly surprising since some historians take a view that *any* societal change is due to the advancement of technological tools, be this the invention of printing technology or the invention of wireless communication. However, the novelty is in the kinds of technology and in the speed of the change.

In a summarizing presentation for the group we wanted to look into the future. However, rather than foretell the future – a task that would be impossible considering the rapid changes in technology – Holton in his concluding address "Tertiary mathematics education for 2024" presented a wish for the future. This wish included emphasis on the "creative" side of mathematics, rather than on its "created" side, that is, emphasis on the activity of doing mathematics rather than the focus on the artifacts of such activity of others. Technology appeared to be one of the means to this end. It further included an emphasis on research in mathematics education that will help understand better the learning process and in turn influence pedagogy. "That mathematics is seen to be something that is to be enjoyed and not feared" – was one of the aspirations put forward by Holton. And though this wish was presented in a rather personal tone, there was a sense in the group that it was shared by many.

List of papers presented or distributed for TSG 3
(available at www.icme10.dk)

D'Arcy-Warmington, Anne: Learning to make happy mathematical memories
Engelbrecht, Johann & Harding, Ansie: Taxonomy of online undergraduate mathematics courses
Gorev, Dvora; Gurevich, Irene; Barabash, Marita: How is the efficiency of the computer usage in geometry related to the levels of students' learning abilities?
Hillel, Joel: Trends in the teaching of linear algebra and the role of technology.
Hockman, Meira: Success at all costs or the cost of success?
Holton, Derek: Tertiary mathematics education for 2024
Kannemeyer, Larry: A reference framework for measuring student's understanding in a first year calculus course
Leikin, Roza: Professional dialog, its components and qualities: from graduate research on teaching to an undergraduate teachers program
Liljedahl, Peter: AHA!: The effect and affect of mathematical discovery on undergraduate mathematics students
Lovric, Miroslav: Transition from secondary to tertiary mathematics, McMaster University experience
Luk, Hing Sun: Gap between secondary school and university mathematics
Martinez Luaces, Victor: Teacher training for problem solving and modeling
Nishimori, Toshiyuki: The deterioration problem of university students' capacity to study mathematics in Japan from 1993 to 2003 and a recent inquiry
Paramonova, Irina: Mathematics syllabus innovation in Russia: The Moscow experience
Safuanov, Ildar: Design of the system of genetic teaching of some topics of algebra at universities
Wittmann, Erich Ch.: Learning mathematics for teaching mathematics: The notion of operative proof.
Zazkis, Rina: Representing numbers: Prime and irrational
Zgraggen, Bernhard: Interactive, generic, heuristic and dynamic step-by-step solutions to mathematical problems in the world wide web.

This report has been written by Derek Holton and Rina Zazkis. They are happy to be contacted at dholton@maths.otago.ac.nz and zazkis@sfu.ca, respectively, for further information on the work of this TSG.