TSG 9: Research and development in the teaching and learning of algebra

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Aims and focus

TSG 9 investigated recent developments in the teaching and learning of algebra and provided participants with a forum for sharing and discussing their research endeavors, development projects, and experiences. The teaching and learning of algebra is a difficult area for study because across different countries, and even within countries, what is done in classrooms can be quite different. Against the backdrop of this challenge for international discussions of research and development in school algebra, the research group focused on:

- describing and understanding the variation of algebra in schools across the world; and
- the influence of technological developments on the algebra curriculum.

Session 1: International perspectives on algebra

In order to create common ground for conversations in our study group about algebra across the world, using the internet, prior to the conference participants in TSG 9 had access to three of the public TIMSS-R (Trends in Mathematics and Science Study, R for "repeat") videos: one each from Hong Kong, Japan, and Switzerland. During the first day of activity, researchers from these three countries commented on these videotapes in the context of outlining the nature of algebra in their society. They were joined by a colleague who presented a videotape from a Mexican classroom and members of the organizing team from the United States, the United Kingdom, and Canada. To orient discussion, as participants listened to these presentations, they were asked to consider the following four sets of questions:

- In your context, *what is algebra*? Is algebra seen as a central mathematical topic or as a less important one? Is it viewed as "the language of mathematics" or as a particular topic of mathematical study, or a set of methods to learn or problems to learn to solve? Is it taught as a separate entity or as part of some larger whole? Where would a visitor to your country find algebra in schools and how would the visitor know how to recognize it?
- Is the algebra curriculum in your context in *flux or change*? Are there tensions between what has been done and what reformers propose for school algebra? Are there different approaches to the content? What do you mean by an approach?
- Is instruction in algebra *similar to or different from other mathematical strands* in the curriculum? Is justification or argumentation different in the algebra classroom? If so, why? Are there patterns in the teacher role or in student participation that are peculiar to the algebra classroom in your context? Again, if so, how do people explain these phenomena? Is there a role for practice in the algebra classroom that is different from other arenas of the curriculum?

- In your context, is algebra thought of as *a difficult subject matter* for students to learn? If so, what are the explanations for this difficulty? When do students study this material? Do all students study this material? In your context, is algebra seen as a difficult subject matter to teach? If so, again why? Is students' motivation to study algebra seen as similar to or different from their motivation to study other aspects of mathematics?

Session 2: Theory in algebra learning

The second and third day of activities each focused on a particular topic. In session 2, *Barbara Dougherty* (USA) and *Toshiakira Fujii* (Japan) presented work involving the introduction of algebra to students. Barbara Dougherty presented a quantity-based approach to the use of algebraic symbols in grades 1-3. Discussion around her presentation involved the nature of the symbols that students used in the project. Toshiakira. Fujii advocated for the use of problems involving arithmetical identities that can lead to the use of numbers as quasi-variables. He suggested that students can come to use numbers as particulars that represent more general constructs, and they can identify and discuss algebraic generalisations long before they learn formal algebraic notation.

Session 3: Roles of technology in algebra curricula

In session 3, *Michèle Artigue* (France) and *Michal Yerushalmy* (Israel) gave presentations on the use of technology in algebra curricula. The presentations from these two authors related to each other closely. On the one hand, Michal Yerushalmy focused on the curriculum/technology nexus. She argued that there are key transitions in the nature of students' algebraic activity as they study algebra that can be used as a window into understanding how technology does or does not transform the algebra curriculum. She concluded that:

> "technologically-supported curricular change can lead to change in students' cognitive hierarchies, though such change may have as much to do with curriculum as it has to do with technology."

In a related vein, Michèle Artigue argued that whatever context in which students work algebraically, researchers must pay attention to the technologies with which they work. For example, she wrote:

> "the notion of algebraic literacy cannot be defined independently of technological considerations. It cannot be considered as something absolute and independent of technology. Each technology shapes algebraic thinking and activity in a specific way which depends on its affordances, constraints and limitations; each technology imposes specific mathematical needs, and a specific intertwining of mathematical and technological knowledge. Each technology shapes what has to be learnt in order to be algebraically literate and how it can be learnt."

Session 4: Short oral presentations and discussion

The final day of activity began with short presentations from colleagues from three more countries. *Charita Luna* (Philippines) presented a study in the context of College Algebra. *Jean-François Nicaud* (France) presented APLUSIX, a computer system for feedback on

symbolic algebraic work. *Martin van Reeuwijk* (Netherlands) presented a game environment for the solving of equations.

The second half of the final session was devoted to a discussion with Michèle Artigue, Barbara Dougherty, and Michal Yerushalmy based on questions written by participants at sessions 2 and 3 in reaction to the presentations. The organizers took these questions and developed the following set of questions. Quick summaries of the discussions of each question are included.

1. *The emphasis in the presentations has been on the problem domain /use of technology. In what sense is this independent of the teacher?*

None of the authors feel that the problem domain or the use of technology is independent of the teacher. Barbara Dougherty reminded the audience that she is the curriculum developer and teacher in the project she described. Michal Yerushalmy and Michèle Artigue both underlined the importance of the institutional role of the teacher and the importance of understanding this role.

2. *What is an "approach to algebra" and can different approaches be combined?*

Michal Yerushalmy in particular called for researchers to be clearer about what they mean by an approach, whether it is a mathematical change to the curriculum, a change in pedagogy, or other changes. There was much discussion about where there is value in using terms like "approaches as a way to speak in a simplified manner to practitioners and policymakers about potential changes to the algebra curriculum. Michèle Artigue asked the audience to consider what is possible to change in a culture and to think about some work of researchers as attempts to "act on a culture." In her paper, Michal Yerushalmy also argues that research on curriculum might focus on the hypotheses that are made when a curriculum sequences student engagement in algebra in particular ways. Such research would be less focused on the effectiveness of a particular curriculum and more focused on the evidence supporting or undermining the hypotheses on which the curriculum is based. Barbara Dougherty's experience with her curriculum suggests that she needs to follow her students beyond fifth grade to continue to support the experiences they have had with the approach they have learned in earlier grades.

3. *If technology is not a magical wand for making the teaching of algebra easier, why should we endure the pain of change, why is it necessary and useful?*

Michèle Artigue argued against the assumptions in the question itself. Her paper suggests that technology is present whenever algebra is done. Martin van Reeuwijk suggested that technology may change who learns algebra. Ros Sutherland also suggested that some of the work on early algebra, like Barbara Dougherty's, might make work with computer technologies with older children less painful.

4. *How might the research on technology in algebra feed back into and inform more standard algebra teaching and bring new insights?*

Michèle Artigue suggested that technology can be useful for researchers by making changes in everyday classroom interaction. Such changes might allow researchers to see algebra classrooms from a new perspective and thus give insights that might not occur otherwise, not only into the use of technology, but also into standard algebra teaching.

She illustrated this contention with a number of examples of how computer algebra system (CAS) use brings in new forms of expressions that are potential windows on mathematical meaning. In her research, students' interactions with these new forms of expressions indicate they do not see the purpose of factorization. Such insights then have potential ramifications in classrooms not using CAS technology.

This report was elaborated by Daniel Chazan and Eugenio Filloy They will be happy to be contacted at dchazan@umd.edu and smmeef@aol.com respectively, for further information on the work of this TSG.