

TSG 14: Innovative approaches to the teaching of mathematics

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Aims and focus

The topic for this study group was both broad and deep: what is innovative for one teacher may be a regular way to teach for others, so as well as providing a showcase for new practices the group had to think about what ‘innovation’ really means. For example, recent interest in ‘typical’ Japanese methods could lead to practices which are innovative in North America, but are clearly standard in Japan. The organisers therefore decided to accept as ‘innovative’ ideas from all over the world which were novel for those who chose to write about them or make presentations about them. The organisers’ intention was that the meetings at ICME would provide opportunities to reflect on the underlying issues of innovation, while also providing opportunities to learn more about what counts as innovative in a range of countries.

To frame our thinking about innovation, presenters were invited to represent the state-of-play internationally. In addition, refereed papers which had been submitted to the TSG were presented by distribution on the associated website. These were written from many perspectives, and there were tensions between maintaining an overall coherence of thinking about innovation and ensuring that the contributions were truly international. It was decided to keep the breadth which represented the full range of interest in the topic and to provide frameworks and discussion at each end of the programme to ensure meta-issues were also on the agenda. The accepted papers generated five subcategories of innovation:

- New ways to engage students affectively in mathematics
- New ways in which learning might take place
- New teaching methods
- Introduction of new topics and contexts into the curriculum
- Use of new technologies

The organisers posed a suite of questions for participants before the group met:

What questions, doubts and resolutions arise for you after reading one or more of these papers?

Is it the case that all these papers indicate improved, or different, learning as a result of innovation? If not, what has changed?

Are the methods and ideas presented here usable in all contexts, or only some contexts? What would be required for an innovation to be of generic usefulness or effectiveness?

Session 1

In the first session, *Laurinda Brown*, UK, presented ways of chanting using a Gattegno number grid. This acted as a timely reminder that unison chanting is not necessarily a mindless activity in classrooms, and that *what* is done is sometimes less important than



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how it is done, and how it engages the learner, consciously or unconsciously, in mathematical structure. *Gary Flewelling*, Canada, then introduced the thoughtful distinction between the 'knowledge game' and the 'sense-making game', showing that many practices which are claimed to be innovative are still concerned mainly with getting learners to acquire static knowledge rather than to become mathematically active, constructing and sense-making for themselves. He demonstrated how an intriguing geometrical dynamic display can be reduced to a sequence of instructions and closed questions, or presented as an arena for supported exploration. Assessment regimes influence the goals of teachers and of learners by focusing on knowledge reproduction. Modelling activities provide opportunities for learners to make sense, both of mathematics and of the context being modelled, and *Sol Garfunkel*, USA, presented examples of how this approach is successful in motivating exploration of mathematics with classes of learners who might not normally engage with the subject.

Session 2

In the second session, *Emily Shahan* and *Megan Staples*, USA, presented video research of classrooms in which teachers were encouraging a problem-solving approach, focusing on the importance of the dialogue between teachers and students. In their presentation, and their accompanying papers, it was clear that the quality and focus of interaction is crucial to the mathematical engagement of learners, rather than the problem-solving situation on its own. Teachers varied in their abilities to maintain collaboration, to get learners to ask questions and make connections for themselves, and to develop and use 'common ground' in their classrooms. The creation of a learning community was also reported by *Binyan Xu*, China, on the website, and *Luo Qiu Ja*, China, also discussed the importance of open, encouraging, interactions.

Anne Watson then presented two tasks, both of which offered opportunity to explore and act mathematically through generating data, conjecturing, and generalising. One of them was about turning a line of cups over in pairs and it was hard to see how this connected with any other mathematics, whereas the other involved school geometry and invited investigation using trigonometric formulae, calculus and geometric proof to find the largest quadrilateral which can be made with sides 7,8,9 and 10 units. This concern to generate a high level of mathematics content through investigation was also mentioned in a website paper by *Xu Liquan*, China,.

These two presentations illustrated the earlier suggestions that the kind of teaching is less important in the teaching and learning of mathematics than the quality of the classroom interaction, the active engagement of learners in sense-making, and opportunities to explore within mathematics as well as in other motivating contexts.

Marcos Cherinda then showed a glimpse of what is possible with a cheap paper weaving board, and the use of weaving as a context for exploring algebraic structure through two-dimensional pattern creation, symbolic representation and prediction. Particularly impressive was the way in which weaving could be used throughout school to create and express appropriate generalisations. Rather than doing 'weaving and maths' once or twice in their school career, learners used it regularly as a generic learning tool.



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Session 3

The value of irregular use of motivating contexts was emphasised in the third session by *Joaquin Gimenez*, Spain, who presented a colourful and varied record of how Spanish learners are encouraged to relate their school learning to situations out of school which can be viewed mathematically. Methods included the use of mathematics trails, real (not 'realistic') problem-solving, liaison with industry and other employment, physical and mathematical model-making, and visits to fairground roller-coasters. The mathematics involved in many of these out-of-school links was not trivial and could include not only complex calculations but also geometrical, statistical and mechanical concepts. Clearly one issue which is important whenever innovative methods are imposed on teachers is the nature of their own mathematical knowledge and the experience they need to make the most of the method, rather than relying on the method to be automatically effective.

This issue was highlighted strongly in the presentation of *Sonoko Mori*, Japan, who told us how in Japan ICT is being incorporated into the mathematics curriculum. Latest data were given and there was a detailed description of problems to be faced, in particular, the training of all mathematics teachers in the use of ICT. The scale of the associated inservice education needs was obvious in this case, but in many cases of imposed innovation within mathematics teaching the inservice education needs of teachers are not obvious at all and new methods which work well with the teachers who created them can be ineffective for teachers who may not recognise the essential features.

The final presentation in the third session was from *Jogsoo Bae*, South Korea, who showed how the use of play materials and sweets, presented to young students by a clown, would motivate them to engage in mathematics. Children who wanted to demonstrate their arithmetic using the materials had to don clown outfits themselves. Bae showed many examples of activities in which he had used the affective and physical drives of children to get them to experience mathematics, such as pacing out huge shapes outdoors, making large models in groups, using colourful and textured materials, and so on. These are in contrast to working formally from textbooks or only with writing materials and the students clearly enjoyed themselves more than in their normal lessons.

Session 4

In the final session of this group, *Wong Khoon Yoong*, Singapore, offered the use of a 'multi-modal thinkboard' as a teaching, learning and assessment medium. The board is divided into several sections radiating from a central topic, and each section is for expressing an example in a different representation, such as diagrammatic, contextual, verbal, symbolic, practical, numerical, graphical and so on. The use of the board could act as a reminder for teachers to provide experience in all modes, as well as a tool for learners to develop complex and interconnected concept images. Use of multiple representations was also mentioned by *Malcolm Swan*, UK, on the website, and he also writes of the need for discussion and reflection to support sense-making. The think-board could scaffold the connections between experiential and formal mathematics which are hard for learners to make without pedagogic input. Indeed, one of the many unresolved questions raised in the final discussion was about how one builds bridges between the learners' experience in any mathematical activity and the growth of procedural, adaptive,



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formal and conceptual understanding which is the goal (in various combinations) of education.

Finally, *John Mason*, UK, brought several strands together by characterising learning and doing mathematics as making mathematical sense of phenomena, some of which may be normally experienced as mathematical, as in the case of the 'largest quadrilateral' problem, and some of which are not, as in the case of the weaving. His view was that it is the action of the learner which makes the phenomenon interesting and mathematical, not the task or phenomenon itself.

There were further papers on the TSG 14 website, www.icme10.dk. These included two papers from *Cecile Ouvrier-Bufferet*, *Denise Grenier* and *Karine Godot*, France, about ways of teaching in which learners research mathematics for themselves. Bufferet's work shows that learners can develop conceptual understanding by constructing their own definitions of concepts, and Grenier and Godot show that learners can, in carefully structured situations and with scaffolding materials, learn how to research mathematics for themselves.

Taking into account points made in the final discussion, and taking all the papers and presentations into account, the group had considered the following matters:

- a range of 'innovative' approaches can be used to engage learners' interests, emotions, physical drives and out-of-school knowledge, in learning mathematics
- there is a tension between the power of intriguing contexts to attract learners to mathematics and the features of the context dominating the learners' experience
- interest, engagement and mathematical activity are generated by the way a learner engages with a task; they are not intrinsic to a task or approach
- the quality of teacher-generated interaction in classrooms makes a difference to how learners engage with mathematics
- all ways of teaching mathematics can be turned into either knowledge-focused approaches or sense-making approaches by teachers, or by curriculum and assessment regimes
- there are tensions between learning to apply mathematics, achieving fluency and achieving understanding; it is not the case that any of these should always dominate, or always precede the others
- it is possible for learners to explore mathematics as if they are researchers, and to access higher-level ideas than those they are currently taught
- there is a need to know more about how to connect sensory experience of mathematical ideas, situated problem-solving, procedural mathematics, conceptual understanding, abstract mathematics, and understandings of structure
- in all innovative approaches the goal of giving students the opportunity a love of mathematics must be a crucial issue.

This report was written by Claudio Alsina and Anne Watson. They are happy to be contacted at claudio.alsina@upc.es and anne.watson@edstud.ox.ac.uk respectively, for further information on the work of this TSG.