TSG 15: The role and the use of technology in the teaching and learning of mathematics

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Introduction

Computer-based opportunities to connect and interact in a variety of ways with representations of mathematical knowledge continue to develop at an extremely rapid rate. As a result, numerous changes can be identified in the way we do mathematics and in the mathematics that we do. The rapid development of both hardware and software has also been accompanied by several educational trends, including increasing integration of technology into many different sectors of the educational world, a range of initiatives for the implementation of digital technologies into mathematics classrooms in different countries across the world, the design of a variety of environments to support a more experiential approach to mathematics learning and the search for new theoretical tools and methodologies that illuminate the learning processes associated with the presence of technology.

Each of these trends can be associated with a variety of questions for mathematics educators, related to issues such as the impact of ever-evolving tools on mathematical cognition and practices, understanding and evaluating the pedagogical approaches and classroom organizations that can be employed in technology-integrated environments, the design of tools that foster mathematical thinking, the epistemological impact of particular forms of representing and communicating mathematics and the challenges inherent in combining technological possibilities with curriculum demands.

Aims and focus

Within the complexity and diversity of this context, it was the intention that the TSG 15 would both serve as a forum in which mathematics educators could come together to discuss and to probe the major issues associated with the integration of technology into scenarios associated with mathematics teaching and learning and as a place to share ongoing work and perspectives. To maximise participation, the TSG programme was developed around: keynote addresses from *Richard Noss*, UK, *Luis Moreno-Armella*, Mexico, *Abigail Lins*, Brazil, *Federica Olivero*, Italy and *Nick Jackiw*, US; poster presentations; software demonstrations; and theme-based discussion groups. Contributions were invited to three inter-related topics.

- *Mathematical thinking, technology and the evolution of mathematics* – clarifying the reciprocal relationships.
- *Orchestration of mathematics teaching in the presence of technology* – understanding structure in the variation.
- *Key factors in the design of new technologies* such as classroom networks, new actions, new representations, and new devices, and the implications of these design factors.

The last of the four sessions was dedicated to a synthesis of the outcomes of the various activities, with reports from the group discussions and a summary of the major issues raised in the plenary activities, posters and demonstrations. A total of 54 contributions were accepted for inclusion, in some form or another, in the programme (available at the TSG 15 site at www.icme10.dk). The contributions brought a diverse variety of perspectives and interpretations. Various different technologies were covered, with nearly – but not all – participants interpreting technology as related to some kind of electronic digital instrument, be this hand-held calculators, stand-alone computers, display devices, class-based networks or aspects of the worldwide web. Through the issues raised by the keynote speakers, interactions during the poster and software demonstrations sessions and the group discussions, the three topics that had formed the basis for the contributions became transformed into three themes around which the outcomes of the topic group can be summarized: *the challenges of research in a constantly changing field, the co-evolution of mathematical knowledge and tools in activity* and *the need for a greater focus on teachers and teaching.*

The challenges of research in a constant changing field

Many of the contributions focused on challenge for both research and practice in an area changing at an exponential rate. As hardware and software evolve, it is not also easy to determine exactly which research questions and issues will disappear from view and those issue which will continue to be important when (and if) we reach the moment to look back on the technological revolution. The discussion around the use of the dragging facilities in dynamic geometry systems (DGS) illustrates various facets of this challenge and serves as a case in point. *Federica Olivero*, Italy, described in her presentation a range of different dragging modalities and how these might mediate the construction of proofs. In particular, in perhaps the most well known modality, the "drag-test", a figure "passes" if certain initial proprieties are preserved as it is dragged around the screen.

In early versions of DGS, the available construction tools were limited to those modeling the ruler and compass constructions of Euclidean Geometry, hence a figure which passed the drag-test, implied a construction procedure that would be valid in 'ruler and compass' geometry. *Gabriel* and *Andreas Stylianides*, USA point out that this is no longer the case in more recent versions of DGS. It is perfectly possible to build a construction to trisect an angle using the calculator tool, for example. Does this mean that previous research findings have been redundant with the changes to the software? Does it mean that we should refine the findings, distinguishing as the Stylianides' do between different types of figure? Or should we accept that the notion of figure cannot be separated from the medium in which is represented? On a slightly different note, when we change our lens from software and learner in research settings, to the mathematics classroom, *Abigail Lins*, Brazil, argued, we cannot assume that the drag mode – let only the drag-test – will be an essential feature of DGS for all users, and especially not for teachers with a history of mediating learning through more static representations.

Given, then, that we are in the still in the midst of the information revolution, *Nick Jackiw*, USA, argued in his keynote talk that the long term value of the work we are doing today may not be in terms of assessments of what will make for effective use and in enumerations of "do's and don'ts", as much as in documentations of technology-mediated change where and when we see it. He pointed to a series of evolution in research into technologies impact on mathematics over the past 25 years. Early research focused on the individual doing mathematics with software has gradually given way to research attempting to recognize the role of the teacher and of curriculum demands on the learner. More recently, it is being recognized that greater emphasis should be placed on the need to understand the mathematical practices that emerge in complex, self-organising, interacting systems, involving multiple learners and teachers using technological tools within and across a variety of settings.

While the first decade of research considered the potential of computers to transform the learning and the teaching of mathematics, the second decade has been characterized by a focus on how technologies also transform the mathematics that is learnt. An important contribution to the field of mathematics education as a whole has been the recognition that in addition to investigating the ways in which the tool, in the course of use, shapes the learner – the instrumentation process – we should also examine the complementary instrumentalisation process, by which communities of users can also shape the tool and hence the setting within which the interactions occur. These reciprocal relations emerged into a second theme discussed during the topic group meetings.

Co-evolution of knowledge and tools in mathematical activity

Luis Moreno-Armella, Mexico, stressed that the phenomenon of the co-evolution of knowledge and tools is not limited to digital technologies, but rather a characteristic of human development. By adopting a historical perspective, he focused primarily on the representational affordances brought by different notation systems which significantly altered the development of mathematical thinking and have become part of today's mathematical infrastructures. Looking back over the more recent history and present day use of digital technologies in mathematics education, various other contributors also provided examples of the ways in which technology shapes and is shaped by learners' mathematical activities. In this vein, both Luis Moreno and *Richard Noss*, UK, described the use of the qualifier "situated" in their attempts to develop theoretical frameworks. Moreno understands by "situated proofs" those expressed in terms of observation and actions permitted by the particular tools of an expressive media. Likewise, Richard Noss used the term "situated abstraction" in his illustrations of how tools and the ways in which they can be used within particular social systems represent an integral part of an individual's evolving conceptualization of mathematical knowledge.

Alongside the aspects of technology linked to its representational infrastructures, *Jim Kaput*, USA, brought into focus the communicational affordances of digital technologies. With advances in connectivity, he described how it is becoming possible for learners to interact alongside computational agents as well as other learners in mathematical explorations, bringing a new layer to what we understand by an experiential approach to learning mathematics – and another possibility with both epistemological and cognitive repercussions. The focus on representational and communicational affordances, like the examinations of the reciprocal relationships between tools, knowledge and thinking, to a certain extent however still leaves to one side what Jim Kaput referred to as the institutional infrastructures (schools, assessment systems, teacher education systems, curricula, etc.). The huge mismatch between the rate of change to representational infrastructures and to institutional infrastructures, respectively, motivated the third theme for discussion.

The need for a deeper understanding of teaching in the presence of technology

One unanimous point that emerged during the conference was a need for still more research that places the teacher as central focus. *Gail Burrill*, USA, reporting on the discussion of the on of the theme-based discussion groups (the group had focussed on teachers and technology), expressed the overall feeling that digital technologies have as yet made little systematic impact on mathematics as it is experienced in the great majority of the world's classrooms. During the third and fourth decade of research in this area, it is recommended that this area is given priority in research. As Abigail Lins pointed out, it may be more fruitful in future research to stress affordances, be they representational or communicational, in terms of the relationship *user-technology* rather than as a feature of the software itself, since her research suggests that teachers do not necessarily appropriate all the affordances attributed in the research literature to particular software environments. The process of instrumental genesis, by which an artefact becomes an instrument, presented by *Cristina Sabena*, Italy, was discussed as one theoretical approach useful in understanding the complexity of appropriating technology into practices. However, even within this framework research endeavours to date have concentrated far more on the integration of technology into mathematical practices than on its appropriation in teachers' pedagogical practices.

And here we can identify a somewhat paradoxical situation. Eight years ago, in summing up the discussion of the ICME 8 study group on computer-based learning environments, Jim Kaput predicted a continuing transition from "Doing (old) Things Better" to "Doing Better Things". The recognition of the transformation of mathematics by technology that permeated the TSG 15 discussions suggested that many participants are committed to the latter. However, there remains a problem with the mathematical legitimacy associated with the so called "better things". Tomorrow's technology might permit a mathematical discourse that differs substantially from that of today's curriculum, but unless changes in the institutional infrastructures accompany the changes in the representations and communication patterns supported by new technologies, the role of technology in school mathematics may continue as peripheral rather than central.

Postscript

Sadly ICME-10 will be *Jim Kaput*'s last ICME. He died in August 2005, following a traffic accident whilst out jogging near his Dartmouth home. Jim was a tireless contributor to the area of technology and mathematics education. He will be remembered, amongst many things, for his visionary insights, his enthusiasm in the face of innovations and his commitment to bringing a powerful and meaningful mathematics to all mathematics learners, exemplified in his work on democratising access to the mathematics of change. Characteristic to Jim's approach, it was his intention that the discussion during the TSG 15 meetings at ICME-10 would contribute to informing the longer term view on technology in mathematics education. This desire is evident in the way his own research programme has examined the new mathematics that particular technologies make possible, both from a historical perspective (looking back) and with an eye to future developments (looking forward). During the TSG meetings, Jim shared aspects of his recent work on classroom connectivity, fitting as he himself had a great gift for making connections – one of the reasons for which he will be missed by so many of us.

This report has been written by Lulu Healy. She is happy to be contacted at lulu@pucsp.br for further information on the work of this TSG.