TSG 17: The role of the history of mathematics in mathematics education

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Topic Study Group 17

Aims and focus

The aim of TSG 17 was to provide a forum for participants to share their teaching ideas and classroom experience in connection with the history of mathematics, in the spirit of the 10th ICMI Study on the role of the history of mathematics in the learning and teaching of mathematics (of the ICMI Study Volume titled *History in Mathematics Education: The ICMI Study*, edited by John Fauvel and Jan van Maanen, published in 2000), and to learn about work that has been done since then.

Roughly put, there are three aspects, which constitute closely related and yet separate issues:

(1) Doing research in the history of mathematics,
(2) Teaching the history of mathematics,
(3) Integrating the history of mathematics in the teaching of mathematics.

The four sessions in this group focused on aspect (3), in an effort to make clearer the meaning of a *historical dimension in mathematics education* and to deepen the understanding of its various aspects.

Programme

In addition to invited contributions there were also submitted contributions, which went through reviews. The final programme included 12 presentations scheduled in four one-hour sessions: 5 invited talks, 4 oral presentations and 3 presented by distribution contributions. Each presentation was followed by discussion among participants. In the last session an extra half-hour was devoted to a general discussion and a summary of the main points raised during the four sessions. Relevant material on the presentations has been made available on the TSG 17 section of the web page in the form of extended abstracts, full texts, related papers, or links to other web sites. Prospective participants were able to download material of interest to them and study it in advance. However, hard copies of material on some presentations were also available at the meeting. At least 64 people from 22 countries participated in this group.

Invited talks:

Chun-Ip Fung, Department of Mathematics, Hong Kong Institute of Education, China: "How history fuels teaching for mathematising: Some personal reflections"

Fulvia Furinghetti, Department of Mathematics, University of Genova, Italy: "History and mathematics education: A look around the world with particular reference to Italy"

Michel Helfgott, Department of Mathematics, State University of New York at Oswego USA: "Two examples from the natural sciences and their relationship to the history and pedagogy of mathematics"

Jan van Maanen, Department of Mathematics, University of Groningen, The Netherlands: "History in mathematics education: FAQ and facts"

Guillermina Waldegg, Departamento de Investigaciones Educativas, Centro de Investigación y de Estudios Avanzados del IPN, Mexico: "Problem solving, collaborative learning and history of mathematics"

Oral presentations:

Giorgio T. Bagni, Department of Mathematics, University of Rome "La Sapienza", Italy: "Prime numbers are infinitely many: Four proofs from history to mathematics education"

Marita Barabash and *Raisa Guberman-Glebov*, Achva Academic College for Education, Israel, "Seminar and graduate project in the history of mathematics as a source of cultural and intercultural enrichment of the academic teacher education program"

Daina Taimina, Deparment of Mathematics, Cornell University, USA: "History of mathematics and mechanics in the digital Reuleaux kinematic mechanism collection"

James Tattersall, Department of Mathematics, Providence College, USA and *Shawnee L.McMurran*, Department of Mathematics, California State University, USA: "Using the "Educational Times" in the classroom"

Papers by distribution:

Richard J. Charette, Central Connecticut State University, USA: "Integrating the history of mathematics in the teaching of mathematics"

Constantinos Tzanakis, Department of Education, University of Crete, Greece: "The ontogenetic development parallels the historical development: To what extent is this claim true, or false? Remarks and results from some case studies"

Oleksiy Yevdokimov, Kharkov Pedagogical State University, Ukraine: "Using material from the history of mathematics in learning by discovery"

Summary

Introducing a *historical dimension in mathematics education* involves three different areas: *mathematics*, *history* and *didactics*. Implicit in the presentations and discussions in this group was the key issue: to find and elaborate on a harmonious, balanced and effective interrelationship among these three scientific areas in a way that is enlightening and fruitful in mathematics education. More specifically, out of the discussion it becomes clear that the following two points are most needed in this context:

(i) There is a need to construct and develop appropriate relevant *didactical material* which can either be used directly in the classroom or constitute a resource for mathematics teachers. The material should aim to motivate and guide the teacher to improve the teaching approach or understand better students' difficulties or their idiosyncratic ways of learning mathematics.

(ii) There is a need to enrich *teachers' education* at all levels in this direction, both by introducing courses in (particular aspects of) the history of mathematics and its relation to other disciplines, and by letting them become acquainted with historically inspired material that can be, or has been, used in the classroom. In this way, teachers may hopefully begin to think of a historical dimension in teaching as a possible path for improving mathematics education at all levels, and may develop confidence and trust in this endeavour.

In this perspective, the presentations in this group can roughly be classified, as follows:

(1) *Presentations focusing mainly on introducing a historical dimension in mathematics teachers' education*

a) *Reports on specific courses in teachers' training*: *Waldegg's* presentation concerned junior high school mathematics teachers' collaborative work on problem-solving, based on historically motivated themes, using for that purpose excerpts from *The History of Mathematics* (Nuffield Foundation 1994). *Barabash* and *Guberman-Glebov* reported on a sequence of activities in prospective mathematics teachers' education that aims at making history an integral part of students' education programme and allowing them to profit from this knowledge in their teaching practice through the design and implementation of particular teaching units.

b) *Presentations reporting on the design of didactic material or its implementation in practice. Tattersall* and *McMurran* reported on the use of the (recreational) mathematical problems published in the *Educational Times* in the Victorian era. They talked about the origin of this material and gave a sample of examples they use in their teaching. *Taimina* reported on the rich didactic material (still under development) that comes out of a kinematic mechanisms collection developed by F. Reuleaux in the 19th century and its possible use to unfold the underlying deep mathematical ideas, concepts and methods emphasizing as well the relation between mathematics and mechanics. *Yevdokimov* presented some examples from an e-learning textbook on Euclidean geometry that elaborates on historical problems and contains related historical information.

(2) *Presentations focusing on integrating history into classroom teaching.*

Three speakers presented particular examples and the underlying rationale, aiming to illustrate how history may contribute to the improvement of mathematics teaching in one way or another – by exciting the students' interest, enriching their view of mathematics or deepening their awareness of what mathematics really is. *Fung* gave two examples enlightened by historical materials to illustrate the point of view that it is essential in mathematics education to design and investigate what E. Wittmann calls "substantial learning environments", where students are engaged in the process of mathematising (in H. Freudenthal's sense). His talk was accompanied by a short video on classroom activity, which was particularly illuminating. Through historical examples in optics (contributions of Heron, Fermat, Leibniz and Huygens to geometrical optics) and chemical kinetics (Briggs and Haldane's steady-state hypothesis) *Helfgott* illustrated the deep interrelation between mathematics and the physical sciences, and how rich and fruitful teaching ideas this can generate *Charette* outlined historically motivated teaching capsules on elementary geometry. One more paper by distribution came from

Alejandro R. Garciadiego, which is on how the history of modern, advanced mathematics, in this case the concept of a well-ordered set, can be illuminating in the context of undergraduate teaching. Unfortunately, the presentation was cancelled because the author did not participate in the Congress.

(3) *Presentations focusing on more general issues:*
van Maanen talked about questions from different quarters asking about the role of history in mathematics education, and classified them according to whom the enquirers are, what they ask and what can be the feedback from such questions to all those interested in integrating history into mathematics education. *Furinghetti* presented an outline of the different views of the role of history in mathematics education and identified two main lines of approach: (i) history as a vehicle to reflect on the nature of mathematics as a socio-cultural process, including the idea of history as a means to promote mathematics in the classroom in order to humanize mathematics; (ii) history as a possible way to conceive and understand mathematical objects, thus referring to the core of problems related to teaching and learning mathematics. Both these broad lines of approach include attempts to answer key questions like: "For a teacher or for a student, is it advisable to know the history of mathematics? How much history does one have to know? And how does one have to know history?" *Bagni* discussed some epistemological issues related to the historical analysis of a mathematical topic, necessary for achieving an effective and correct use of historical data in mathematics education, and presented some theoretical ideas that underline the prime importance of a correct social and cultural contextualisation. He used as an illustration different proofs of the infinitude of prime numbers offered at different 'historical periods' and in different mathematical settings. *Tzanakis* considered the quite old, but still unsettled, question of whether and to what extent the ontogenetic development parallels the historical development in mathematics? What kind of analogies are observed, and what can mathematics education research profit from investigating such analogies? The general ideas presented were supported by comparing the historical development with data obtained from empirical research in three case studies: (i) the order relation on the number line, the algebra of inequalities and the concept of the absolute value of a number (ii) the concept of plane in Euclidean geometry, and (iii) the introduction of basic statistical concepts and relations.

Full texts of these presentations were collected and refereed. Accepted papers appeared in a special double issue of the *Mediterranean Journal of Research in Mathematics Education*, vol.3, No1-2, 2004, published by the Cyprus Mathematical Society. During the final discussion two useful resources were mentioned: (i) a new online magazine of Mathematical Association of America (edited by V. Katz and F. Swetz) called *Convergence* (http://convergence.mtahdl.org) where mathematics, history and teaching interact, (ii) a forthcoming publication in a CD version from the Mathematical Association of America titled *Historical Modules for the Teaching and Learning of Mathematics* (edited by V. Katz and K.D. Michalowicz). Such resource materials will be of great interest to mathematics teachers who are enthusiastic about activities discussed in this group.

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Final remark

History of mathematics is not to be regarded as a panacea to all pedagogical issues in mathematics education, just as mathematics, though important, is not the only subject worth studying. It is the harmony of mathematics with other intellectual and cultural pursuits that makes the subject even more worth studying. In this wider context, the history of mathematics has yet a more important role to play in providing a fuller education of a person.

This report has been written by Man-Keung Siu and Constantinos Tzanakis and was approved by all the team members. The authors are happy to be contacted at mathsiu@hkucc.hku.hk and tzanakis@edc.uoc.gr, respectively, for further information on the work of this TSG.