TSG 18: Problem solving in mathematics education

Team Chairs: *Jinfa Cai*, University of Delaware, Newark, USA
Joanna Mamona Downs, University of Patras, Greece
Team Members: *András Ambrus*, Eötvös Loránd University, Budapest, Hungary
Hideki Iwasaki, Hiroshima University, Japan

Aims and focus

The general aims of the TSG 18 were to provide a forum for those who are interested in any aspect of problem solving research at any educational level, to present recent finding, and to exchange ideas. The primary concerns were: (1) To understand the complex cognitive processes involved in problem solving; (2) To explore the actual mechanisms by which students learn and make sense of mathematics through Problem solving, and how this can be supported by the teacher; and (3) To identify future directions of problem-solving research, including the usage of information technology. In the second and third time slots available to the group at the congress, six sub-sessions were organized for researchers around the world to present their new findings.

A more specific aim of the group was concerned with determining the scope of problem solving. We perhaps can discriminate three major categories:

- Problem solving for developing general experience (e.g., non-standard tasks, open-ended questions and project work, modelling 'real-life' situations, setting tasks with impressive solutions for motivation);
- Problem solving specially designed for enhancing targeted conceptual development;
- Problem solving specially designed to stress reflection and valuation of solution paths, and the explicit development of techniques, heuristics and exploratory methods.

Having these three broad perspectives (along with many others that are more local), the organisers raised the question whether it is feasible or useful to talk about a single identity for problem solving. This theme was taken up by a round-table discussion that took place in the first time slot. Also, it is important to develop the field with all these perspectives kept in mind. This motivated us to organize, in the last time slot, two plenary addresses to consider the future directions of problem solving research.

First session: Roundtable discussion

In this session, the roundtable discussion took place, with the theme 'the identity of problem solving'. The rationale was to ask why there exists a particular sub-field called 'problem solving' within the areas of interest of mathematics education, when the phrase 'problem solving' would seem almost synonymous to doing mathematics anyway. Due to the time constraints, this issue was not illuminated much as such, but it provided a perspective that helped in 'coloring' some more specific themes in problem solving that were raised. The panellists invited were *Lucia Grugnetti* (Italy), *Kazuhiko Nunokawa* (Japan), and *Carolyn Maher* (USA). Each gave a short talk on themes suggested by the organizers, followed by questions and comments from the audience. The reactor was *Joanna Mamona-Downs* (Greece). A short summary is given below:

Lucia Grugnetti talked about constructivism in problem solving by describing the 'situation-problems' approach to mathematics teaching currently espoused in France, and talked about the importance of comparing peer solutions for students to appreciate formal explanations. Kazuhiko Nunokawa categorized four objectives in problem solving. i.e., enrichment of schemata, motivating students by exposing them to 'impressive' results, creating personal new mathematical knowledge, and giving experience to enhance general solving ability. In particular, he pointed out that it is important to identify the limitations of using problem solving in teaching whole mathematical theories. Carolyn Maher referred to a long-term project in which the same students were followed from primary school up to upper school. She focused on collaborative work, timely return to problems as students mature, building personal representations, monitoring one's own work. Further she made remarks concerning sense making, as well as affective issues, and the role of the teacher.

Second and third sessions: Presentations of refereed papers

The organizing team had a good response to its call for papers. In total 28 abstracts were submitted to the team before the deadline date, 18 of which were accepted. On the whole these submissions displayed a wealth and diversity of the material incorporated and the decision of acceptance/rejection was sometimes difficult. All authors of accepted abstracts were invited to present their work in a 15-minute talk during the group's sessions at the congress. These talks took place in the second and third time slots with activities being distributed over three different locations. Despite the resulting division of the participants attending the group, the sizes of the audiences were generally satisfactory, and the discussion lively.

Below we shall briefly outline each contribution. The organizing team gathered the papers into collections of two or three that shared some common characteristic. The description follows this grouping into the six ensuing sub-sessions. Full papers are available in the TSG's page in the congress's web site. The following people chaired the sub-sessions: A. Ambrus, M. Downs, H. Iwasaki, S.Leung, E. Pehkonen, and R. Speiser.

Sub-session 1: Broad issues and research projects in mathematical problem solving

The problem–solving agenda incorporates many differing concerns, so it is important to develop insights into how these concerns fit together. The three papers in this sub-session contribute to this in different ways. *Beth Southwell*, Australia, reported on an on-going project to develop a concept map of elements of problem–solving processes and to locate research that throws light on it. *Bernd Zimmermann*, Germany, discussed the use of historical material in various educational aspects of problem solving, such as explaining students' cognitive barriers, individual differences in strategies and the competence of teachers in making diagnoses. *Lucia Grugnetti*, Italy, *François Jaquet*, Switzerland, and *Daniela Medici*, Italy, described a broad international project where collaborative work, the role of designing problems, and the occurrence of unexpected student behavior are stressed.

Sub-session 2: Problem solving in an ICT environment

Researchers in problem solving show a strong interest in applications of Information and Communication Technology, as the following contributions illustrate. *Sergey Rakov*, Ukraine, demonstrated how a Dynamic Geometry package can contribute to problem posing, making hypotheses, finding evidence or counterexamples, and giving approximate solutions. *Ioannis Papadopoulos*, Greece, considered the advantages and disadvantages in using software for teaching the concept of area for primary school children. He pointed out that some activities (such as the cut and paste method) raised by the use of the computer in the end had more of a problem solving nature rather than contributing to conceptualization. *Tom Lowrie*, Australia, talked about a project where students were asked to think about some mathematics implicit in a popular computer game in order to motivate them to engage in mathematical thinking, and to link 'out-of school' and 'in class' activities.

Sub-session 3: Proof, modeling and teaching heuristics

These three themes deal with some specialized but central issues in problem solving. *Keith Weber*, USA, considered the problem solving aspects of forming proof, stressing the difference between obtaining a logical deduction (syntactic) and an argument that is meaningful (semantic). *Zemira Mevarech*, Israel, and *Bracha Kramarski*, Israel, reported a study showing that collaborative work is not sufficient for the enhancement of students' abilities in modelling. In addition special tuition in metacognitive processes is needed. *Murat Altum*, Turkey, and *Cigdem Arslan*, Turkey, gave evidence that teaching targeted heuristics can positively effect students' use of those heuristics.

Sub-session 4: Teacher development and questions of design in problem solving

If we want students to learn and make sense of mathematics, how should teachers design pedagogically sound problems for classroom instruction? The two papers presented in this sub-session addressed this question from experiences in Japan and Singapore. *Takeshi Yamaguchi*, Japan, and *Hideki Iwasaki*, Japan, put forward a view about the pedagogical design of problem solving based on the Dörfler's generalization model. *Kai Fai Ho*, Singapore, *Teong Su Kwang Teong*, Singapore, and *John G. Hedberg*, Australia, presented findings from a survey of 140 Singaporean 5th graders. In this study, students were asked to solve some problems, and to describe the reasons why they had difficulties in solving the problems. It was found that the students surveyed appeared to have limited knowledge of problem solving heurisitics, and the researchers suggested that Singaporean mathematics teachers need to enhance their instructional practices by explicating the processes of mathematical problem solving.

Sub-session 5: Sense-making in mathematical problem solving

Validating results has a long tradition as an important aspect of mathematics education, particularly in relation to problem solving. In this sub-session, *Maria de Hoyos*, UK, examined if and how students seek validations in their problem solving. She found that students do validate their results and do it by constantly seeking what has been called 'mathematical conviction' and 'cognitive reassurance'. *Ban Har Yeap*, Singapore, presented a study exploring how to prevent children from suspending their ability to make sense of mathematics when they perform mathematical tasks such as word problem solving, and, in the process, engage these children in critical thought. *Charita A. Luna*, The Philippines, and *Lourdes G. Fuscable*, The Philippines, examined the impact of mathematical symbolism on college students' problem solving performance. They found that mathematical symbolism has a powerful influence on students' problem solving performance especially in the translation of a word problem into an equation. Familiarity with mathematical symbolism enhances their problem solving ability.

Sub-session 6: Issues in mathematical exploration
In this sub-session, *Vic Cifarelli*, USA, and *Jinfa Cai*, USA, presented a conceptual framework concerning mathematical exploration. In this framework, they view mathematical exploration as a recursive process in which solvers determine goals of action as they formulate their problems, solve them, and reflect upon their solution activities to formulate new problems. They also presented some empirical data to support the conceptual framework. *E. Koleza*, Greece, and *M. Iatridou*, Greece, investigated the role of experimentation in problem solving. They examined and analyzed the mechanisms of experimentation by pre-service teachers, working in four person groups, whilst engaged with mathematical problem solving. *J. Piggott*, UK, presented the key aspects of mathematics enrichment and how the content and design of new resources would continue to build upon them.

Fourth session: Future directions for mathematical problem solving research
Kay Stacey, Australia, and *Edward Silver*, USA, delivered two plenary addresses to discuss the future directions of problem solving research. *Jinfa Cai*, USA, chaired the session. Stacey argued that while teachers around the world have had considerable successes with achieving various goals concerning problem solving, there is always a great need for improvement, so that more students get a deeper appreciation of what it means to do mathematics. This requires research with associated curriculum development directed to understanding the problem solving process for mathematics (in all its aspects), developing effective classroom practices, and designing suitable tasks. Silver argued that the work done to date has helped us gain important insights into how students might learn to solve problems but that it has paid too little attention to ways in which problem solving might be a core element in classroom instruction. He suggested that more work should be done that directly addresses the following central issue of importance to classroom teachers: What do the findings from research suggest about the feasibility and efficacy of teaching mathematics *through* problem solving?

An underrepresented theme
Problem posing is at the heart of mathematical research and scientific investigations. In fact, in scientific inquiry, formulating a problem well is often viewed as even more important than finding its solution. In mathematics education, there is a broad consensus of viewing mathematical problem posing as an essential and effective instructional practice. It is suggested that problem-posing activities not only lessen students' anxiety and lead to a more positive disposition towards mathematics, but also enrich and improve students' understanding as well as problem solving capacity. Given the importance of problem posing activities in both school and college mathematics, in recent years the mathematics education research community has began to investigate various aspects of problem posing processes. Despite the general interest of mathematical problem posing TSG 18 did not have any contributions devoted completely to this very theme. Hopefully, more attention to problem posing will be evident in the presentations at the next congress.

Final remarks

In the last three decades, there has been a great deal of educational research on mathematical problem solving which has deepened our understanding of the field immensely. As we reflect on the research trends on mathematical problem solving, we realize just how dynamic research on mathematical problem solving is. This is hardly surprising, when one considers some of the fundamental questions that the field has to address, such as: What is mathematical problem solving? What are the cognitive processes used in solving mathematical problems? What are the purposes of problem solving? What are the actual mechanisms in which students use to learn and make sense of mathematics through problem solving? What is the teacher's role in implementing problem solving in the mathematics classroom? The views of the mathematics education community on each of these questions have evolved over time and are still in flux. It is appropriate to periodically take stock of the field by examining how mathematics educators are currently looking at problem solving and seeing what issues currently have a need of further research. This TSG group served this purpose well. An indication of this claim is that the editors of the *Journal of Mathematical Behavior* solicited a selection of the work from TSG 18 for publication that resulted in the two special issues (Volume 24, issues 3 and 4).

This report was written by Jinfa Cai and Joanna Mamona Downs. They are happy to be contacted at jcai@udel.edu and mamona@upatras.gr respectively, for further information on the work of this TSG.