TSG 20: Mathematical applications and modelling in the teaching and learning of mathematics

Team Chair: ***S. Kenneth Houston***, University of Ulster, Jordanstown, Northern Ireland
Team Members: ***Toshikazu Ikeda,*** Yokohama National University, Japan
Nikos Klaoudatos, University of Athens, Greece
João Filipe Matos, University of Lisbon, Portugal

Introduction

The focus for TSG 20 was applications and modelling in the teaching and learning of mathematics and the scope included secondary or high school and university and college levels. The aims of the study group were to review the present state of our knowledge of how students learn applications and modelling, to hear from educational researchers of their very recent findings in this field of enquiry, and to discuss teaching innovations and interventions which are intended to enhance student learning.

It is right and proper that this theme should attract so much attention. Mathematics, of course, should be studied as of right for many reasons, but the application of mathematics and the technique of mathematical modelling, and the role these play in the world and in the lives of everyone must be included in the curricula of schools and universities. Furthermore, it is the contention of advocates of applications and modelling that the learning of mathematics itself is motivated, enabled and enhanced through the study of applications and modelling. This is testified to by many research studies, including some of those presented at TSG 20 and reported below. Mathematical modelling is a "way of life" for professional mathematicians (academic pure mathematicians excepted); it is akin to the scientific method employed by scientists, engineers and technologists; engaging in it enhances curiosity, an inquiring frame of mind and key life skills; it is employed in many areas of human endeavour, both social and scientific.

Invitations were issued through ICME-10 channels for authors to submit papers to the TSG. These were to be refereed and those selected would be "presented by distribution" via the TSG. They would be published on the ICME-10 website in advance of the conference and would be reported during the sessions allocated to the TSG. A considerable number were submitted and sixteen survived the refereeing process. These papers were to form the major part of the work of the TSG. The Organising Team agreed to use the first three sessions to report the papers and for brief discussion and to use the final session for further discussion and debate. The topics of the accepted papers fell into three categories – Sharing Good Classroom Ideas, Empirical and Theoretical Studies at Secondary Level and Empirical and Theoretical Studies at Tertiary Level. Rather than invite authors to present their paper in a short time, it was agreed to appoint an expert in each area to summarise and present the group of papers, and to have the authors in attendance to add further comments and to answer questions. This way of proceeding worked very well, the sessions did not over-run and there was lively interaction with the audience.

Details of each of the papers presented are included in the following sections.

Session 1 – Sharing good classroom ideas

Session 1 was presented by *Gabriele Kaiser* (Germany) and had six papers, two at secondary level and four at tertiary level. The two papers relating to secondary level mathe-

matics were written by high school teachers from the USA. They both gave excellent ideas for bringing aspects of the real world into the classroom. *Ralph Farnsworth*'s (USA) paper was on "The Use of Geometry and Proportional Reasoning Techniques at the US Department of Agriculture". The mission of the Farm Service Agency is to help farmers to stabilize farm income, to conserve land and water resources and to help farm and ranch operations recover from the effects of disaster. These ideas were used in the classroom. For children living in rural communities, the agricultural setting of the problems presented by this work would make them very real indeed. Farnsworth gave examples of many situated problems that had all been tried and tested with his grade 10 pupils.

Steve Krevisky (USA) also presented ideas for high school classroom use, but this time inspired by the world of sport. His paper was titled, "The National Collegiate Athletic Association Basketball Championship Tournament – Statistics, Prediction and Analysis". Using recent historical data concerning the results of the NCAA annual basketball tournament, he worked out the relative frequencies of the different possible outcomes of the matches, and used these as predictive probabilities for the future. His pupils enjoyed this activity and it helped them learn some ideas in probability. These ideas could be used in many one-on-one sports such as the major international tennis championship tournaments.

The paper by *Hamid Chaachoua* and *Ayse Saglam* (France) – "Modelling by Differential Equations" – looked at the close relation between physics and mathematics. The historical development of modern physics is littered with models that involve differential equations. However, the situation in France, and in other countries as well, is that, while university students learn to solve the equations, they do not spend much time on the "modelling" that relates the equations to the physics. Thus they are not getting into the "way of life" of mathematical physicists. The paper gave several situations where the authors had observed this behaviour.

Another cross disciplinary paper from the USA – "Stealing from Physics: Modelling with Mathematical Functions in Data-Rich Contexts" – was submitted to this mathematics teaching conference when the author, *Tim Erickson*, and his colleagues realised that their work in physics was also very relevant to the learning of mathematics as well. University students were given data that arise in real world phenomena and they learnt to model these with functions. It was observed that the students of the two different disciplines had difficulties bringing the ideas of mathematics and the ideas of physics together. Erickson suggested that using their resource material and software tools helped students to overcome some of these difficulties, but he recognised the problem mathematics teachers have in finding time to let students carry out experiments.

Otávio Jacobini and *Maria Wodewotzki* (Brazil) used mathematical modelling in the university classroom to provoke some political thinking by their pupils to encourage greater "citizenship". They looked at the income tax laws in Brazil and speculated on how modifications to these might bring about social change. But more than this, they described how their pupils worked in a community project, teaching some teenagers who had committed some crime and were now in a rehabilitation programme. The students, by their peer teaching of the teenagers, also enhanced their own learning.

Finally in this section, from Iceland, *Thorir Sigurdsson* asked the question, "Could a Mathematics Student have Prevented the Collapse of the Atlanto-Scandian Herring?" Simple models to fit data were derived and analysed. This is yet another example of an investigation that is very relevant to the lives of the university students involved.

These six papers are published in *Teaching Mathematics and its Applications*, volume 25, number 1, 2006.

Session 2 – Empirical and theoretical studies – secondary

This session, presented by *Peter Galbraith* (Australia) saw the first airing of the results of six empirical studies, investigating questions of great interest to the community. There were three studies from Germany – *Gabriele Kaiser*'s study on the "Development of Mathematical Literacy", *Katja Maaß*'s study on "Barriers to, and Opportunities for Integration of Modelling in Mathematics Classes", and *Dominik Leiß*'s study on "Teacher Intervention versus Self-Regulated Learning."

Kaiser studied the development of mathematical literacy through an innovative teaching programme that relied heavily on applications and modelling. She used five levels of mathematical literacy, ranging from "illiteracy" – the inability to cope with relevant information, to "multidimensional literacy" – which incorporates contextual understanding and philosophical, historical and social dimensions. Kaiser found that the teaching programme encouraged great progress at the lower levels of literacy, but not so much at the higher levels.

Maaß's study aimed at showing the effects of integrating modelling tasks into the daily school routine. She particularly studied how students' mathematical beliefs changed through the course, and the connection between beliefs and modelling competencies. Beliefs were classified as understanding mathematics to be "process", or "application", or "formalism", or "scheme". Those students who had applications oriented or process oriented belief systems also had more positive attitudes towards modelling. Maaß concluded that modelling examples should be integrated into the early years of education to prevent barriers being raised in later life due to inappropriate beliefs.

Leiß reported that German students generally underachieved when faced with demanding tasks, and that German teachers had difficulties diagnosing and handling students' problems. He claimed that "work on tasks" in the mathematics classroom is all-important for students, and so the selection, design, handling and assessment of these tasks are all-important for teachers. The teacher also has to work out when to intervene and when to let the student work independently. Leiß's conclusions are worthy of study by teachers – students should put themselves mentally into the problem situation; authentic tasks require students to have some specialist knowledge of the situation of the task; students need to reflect upon their solution process, especially looking at all phases of the modelling cycle.

Jerry Legé (USA) in his paper "Approaching Minimal Conditions for the Introduction of Mathematical Modeling", considered two different instructional approaches – behaviourist and constructivist – to introduce modelling to students with weak content skills and no prior modelling experience. The students in the treatment groups were from two different high schools and the course was at pre-algebra level. The tasks related to aspects of contemporary student culture and included "planning a vacation" and "best rap artist". For the students, both treatments worked, but in different ways and both impacted strongly on the heretofore traditional nature of classroom instruction in these schools.

In Taiwan high schools, *Fou-Lai Lin* and *Kai-Lin Yang* told us, the environment is decidedly unfriendly to the teaching of modelling. The backgrounds of teachers and students, the examinations and the textbooks all militate against it. Nevertheless these

authors ventured forth with a teaching intervention involving working on suitable modelling tasks, in an attempt to influence the prevailing culture. The tasks related to situations familiar to the students such as the design of furniture, the location of fire stations and the misuse of drugs. Lin and Yang found that, at the end of the project, students were beginning to exhibit the distinctive characteristics of the modelling process such as verifying the model and going round the modelling cycle.

Allan Tarp (Denmark) presented an interesting discussion paper on the importance of the words we use when teaching modelling. He distinguished between LAB- or laboratory words and LIB- or library words. Thus "Brahe, by observing and recording the motion of the planets provided LAB-data, from which Kepler induced LIB-equations that later were deduced from Newton's LIB-theory about gravity." He concluded that by replacing the authorised LIB-routines of mathematics with authentic LAB-routines solves the "relevance paradox" of mathematics, which comes from the "simultaneous objective relevance and subjective irrelevance of mathematics".

These five papers are published in *Teaching Mathematics and its Applications*, volume 24, number 2-3, 2005.

Session 3 – Empirical and theoretical studies – tertiary

There were four papers in this session, presented by *Chris Haines* (United Kingdom) including one by *Ros Crouch* and himself. Haines and Crouch, writing on "Applying Mathematics: Making Multiple-Choice Questions Work", discussed how some multiple-choice questions may be used to improve understanding, to develop and to assess modelling capabilities and as an aid to teaching. This is a development of ideas that they, and others, have been working on for several years and which is published mostly in the ICTMA series of books (ICTMA, 2004). Each question is designed to look at a single phase of the modelling cycle. These have proved effective in the uses mentioned above.

Djordje Kadijevich (Serbia and Montenegro), *Lenni Haapasalo* (Finland) and *Jozef Hvorecky* (Slovenia) asked pertinent questions about "Using Technology in Applications and Modelling", which is the title of their paper. What implications does the availability of technology have for the nature of the modelling problems that can be given to students? How does its use facilitate learning? When does it enrich learning possibilities? Can we do without it? All good questions, which the authors considered in detail, with examples, and which led them to not unexpected answers.

Thomas Lingefjärd and *Mikael Holmquist* (Sweden), in their paper "To Assess Students' Attitudes, Skills and Competencies in Mathematical Modeling", discussed the successes they had when using various forms of peer-to-peer tutoring and assessment with pre-service trainee secondary school teachers. Students voted 2:1 for peer assessment of their work, realising that it helped develop *self*-assessment. They also found that assessing mathematical modelling was much harder than they had anticipated.

Finally, *Dvora Peretz* (Israel) gave us an interesting novel concept of a model. Her paper is titled "Inverse Mathematical Model – Yet Another Aspect of Applications and Modelling in Undergraduate Mathematics of Prospective Teachers." Using the concept of an inverse model, she presented a useful way of helping students understand and teach ideas in elementary mathematics such as the division of one fraction by another.

These four papers are also published in *Teaching Mathematics and its Applications*, volume 24, number 2-3, 2005.

Session 4 and conclusions

Session 4 was a plenary panel discussion session, the panel consisting of the members of the organising team for TSG 20 and the three session presenters. Many of the issues raised in the papers were revisited and there was lively audience participation.

The organisers believe that TSG 20 worked very well. Fifteen papers by twenty-one authors from twelve countries over four continents were presented. This was a truly international study group. Clearly "applications and modelling" is taken very seriously across the globe.

The ideas presented in session 1 are worthy of replication by teachers at secondary and tertiary levels across the world; they may need some local "customisation" first!

The research findings of the secondary level studies presented in session 2 and the tertiary level studies presented in session 3 provide new insights into how students approach their learning and how the teachers involved managed this learning.

At the end of each of these three sessions and in Session 4, clarifying questions were asked of the authors, and useful discussion ensued. All of those attending expressed their appreciation of the authors and the session presenters for bringing their work to this TSG.

This report was prepared by Ken Houston who has now retired from the University of Ulster but is, nevertheless, happy to be contacted at sk.houston@north-circular.demon.co.uk for further information on the work of this TSG. Contact details for the authors are given in the appropriate issue of *Teaching Mathematics and its Applications*.