TSG 21: Relations between mathematics and other subjects of science or art

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Aims and focus

TSG 21 gathered a group of congress participants who were interested in the "Relations between mathematics and other subjects of science or art". The activities of the group included presentations, a keynote lecture and discussions of new trends and developments in research or practice related to this topic.

Keynote lecture

A keynote lecture was given in the last session by *Monica Wijers*, of the Freudenthal Institute, University of Utrecht, (The Netherlands), who spoke on *Connecting mathematics and other subjects.* In her lecture she referred to the position of mathematics in relation to other subjects according to one of the fundamental principles of the Freudenthal Institute, "Realistic Mathematics Education", pointing out the concern for more stress on connections between subjects in the current Dutch Secondary Education.

While describing the state-of-the-art of this educational level, Wijers referred to: other subjects as contexts, mathematics as a tool and mathematical modeling, showing several examples and reflecting on them. In reference to context use she emphasized the importance of

- not presenting the model but having students choose one and
- using more interesting examples and posing more interesting questions

Monica Wijers further spoke about the risks and benefits of context use, of looking at mathematics as a tool while also describing the corresponding risks and benefits of mathematical modeling. Here she pointed out that that "Emergent Modeling"[2] is a basis for mathematical modeling.

As a final conclusion, she returned to the question: "How to position mathematics in relation to other subjects?" and recommended a careful balance of the risks and the benefits according to the considerations previously exposed.

This lecture turned out to be motivating for further reflection for those present, as was expressed by some participants: "We need something like the track suggested by the keynote speaker".

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1 Joseph Malkevitch participated during the process of organization of the activities of TSG 21 though he could not attend the conference due to personal reasons.

2 Emergent modeling refers to both the process by which models emerge, and the process by which more formal mathematical knowledge emerges. The model develops first as a *model of* the students' situated informal strategies and gradually, as a *model for* more formal mathematical reasoning.
(K. Gravemeijer, 1999. How emergent modeling may foster the constitution of formal mathematics. *Mathematical Thinking and Learning*. **1**(2), 155-177).

The TSG sessions

The group had four sessions which were organized with the purpose of providing both an overview of the current state-of-the-art in the topic and presentations of recent contributions to it, as seen from an international perspective. Specific topic areas of contributions included the following: "Mathematics and Arts", "Mathematics and Sciences", "Mathematics and Interdisciplinarity".

The Organizing Team agreed that the selection of papers for oral presentation would not depend only on the intrinsic quality of the submitted papers but also on the diversity of ideas and domains to be presented in the different time slots. In this line, the following papers were selected for oral presentations:

Mathematics and Visual Arts:
"Mathematics as underlying structure in the arts: A capstone course for preservice teachers", *J.Wanko*, USA

Mathematics and Philosophy:
"Re-creating the Renaissance", *B. Sriraman*, USA

Mathematics and Literature:
"Mathematics, Mathematicians, Literature and Art", *I. Safuanov*, Russian Federation

Mathematics and Music:
"Mathematics and music: some educational considerations", *P. Maher*, UK

Mathematics, Interdisciplinary Competences and Sciences *(with regard to teacher education):*
"Mathematics and science", *P. Baggett* and *A. Ehrenfeucht*, USA
"An Inter Disciplinary Approach To Math Teaching: Mathematical Ideas in Sciences Taught By Future School Math Teachers", *R. Guberman* and *M. Barabash*, Israel

Mathematics, Interdisciplinary Competences and Sciences *(with regard to upper secondary and tertiary education):*
"Interdisciplinary competences integrating mathematics and other subjects of science", *C. Michelsen, N. Glaargaard* and *J. Dejgaard*, Denmark
"Attaining mathematical competences via the use of other subjects in a first year mathematics course at an agricultural university", *T.V. Pedersen*, Denmark
"Courses on mathematical modeling with information literacy: successful attempts at Karlstads' university", *Y. Shestopalov* and *I. Persson*, Sweden
"Cultural astronomy and mathematics in art and architecture: Two general education courses at the National University of Singapore", *H. Aslaksen*, Singapore

These papers covered a wide area of topics relating to the teaching and learning of mathematics and its connections with arts and/or science at secondary, university and pre-service teachers level.

The main conceptions that can be extracted from the different works deal with the deep interrelation between mathematics, arts and science. The arguments for this are founded in the historical influence of mathematics in arts and sciences, in the common aspects that can be found between creative work in mathematics and art, and in regarding mathematics as part of human culture, and as part of mankind's struggle to understand the world.

The educational implication of this interrelation is the need for interdisciplinary activities, to provide a sense of meaning to student's mathematical learning by relating mathematics to different aspects of human culture. In a broad sense these activities might be related to visual arts, music, and literature or to the mathematical modeling of phenomena of life. Different responses to these needs are present in the selected works.

With respect to mathematics and arts, educational activities may include:

- Exploring some mathematical topics in music and visual arts and designing a course for prospective teachers. (Maher)
- Exploring ways in which different mathematical concepts (e.g., symmetry and asymmetry, patterns and randomness, ratios and proportional reasoning) can be identified in the arts and then focusing on the specific mathematics of those concepts (course for preservice teachers). (Wanko)
- Re-creating the Renaissance in a microcosmic way in the high school classroom gives an opportunity for realising and appreciating the underlying unity of the arts and sciences. (Sriraman)

With respect to mathematics and science, educational implications may include:

- Two competencies are to be emphasized: the modeling competence and the problem solving competence using examples taken from applications to train the selected competencies, also taken as a starting point for the presentation of the mathematical theory. (Pedersen)
- The development of meaningfulness in a student's mind by bringing him/her near phenomena of life by introducing inter-disciplinary topics like for example 'From the globe to a map – mapping projections' where mathematical concepts like proportion, similarity, scale, diagrams, axes and coordinates are used. Another example is 'Winds and sea streams' which relates these issues to geometric vectors. (Guberman)
- A program for in-service and pre-service teachers of mathematics. It allows one to combine mathematics and science much earlier that is usually done. (Bagget and Ehrenfeucht)
- Modeling activities are emphasized as an alternative of the traditional transfer method of education, to overcome students' difficulties in combining mathematics and science. (Michelsen et al.)
- Developing *information literacy* (information search in collaboration with others) (Shestopalo and Persson)
- Design of General Education courses based on group projects and homework tasks. Tasks related to astronomical observations are recommended. (Aslaksen)

Participants

Five teaching levels were represented in this group (with overlaps: 20% of the participants teach at primary level, 30% at secondary level, 5% at tertiary level, 70% at university level, while 20% were involved in preservice teacher education).

The poster sessions of ICME-10 included posters related to TSG 21. Many of the authors were active participants at the sessions of the topic study group. Also an active

participant of the TSG 21 sessions was the lecturer of a regular lecture related to arts: *Intersection of mathematics and art* (Vera W. de Spinadel from the International Mathematics and Design Association, Buenos Aires, Argentina).

Inquiry

At the end of the last session a brief questionnaire of three questions was given to the 20 participants. The aim of the questionnaire was to enquire whether the participants included topics relating mathematics with art and/or science in their own teaching, and to get to know participants' personal views about including or not those topics in their classes, and also to detect if there were teachers who would like to do so, without having done it, and which were the reasons in either case. The questionnaire was:

Dear participant, the OT of TSG 21, will be happy to know about you:
Q1. Educational level you work at:
Primary – Secondary – Tertiary – University – Preservice teachers
We would also like to know whether:
Q2. Do you include in your teaching topics relating:
a) mathematics with art? Yes – No – Why? Write three reasons please.
b) mathematics with science? Yes – No – Why? Write three reasons please.
Or:
Q2 Would you like to include in your teaching topics relating:
a) mathematics with art? Yes – No – Why? Write three reasons please.
b) mathematics with science? Yes – No – Why? Write three reasons please.

Three reasons were asked (though this was not expected to be fulfilled completely) to be given in both cases to allow for other reasons than eventual institutional or curricular prescriptions.

Results

Q2: Including topics relating mathematics with arts an/or science in the teaching of mathematics

The responses of the participants showed that topics of science are more often included in the teaching (80%) than topics of arts (65%). These responses can be summarized as follows:

Q2 Mathematics and art	**Q2 Mathematics and science**		
Responses %	YES (80%)	NO (20%)
YES (65%)	50%	15%
NO (35%)	30%	5%

Topics relating mathematics with both arts and science are included by 50% of the participants. In this case the arguments presented referred to:

- Motivation and interest: 20% of the participants in both cases referred to motivation and context:

– "Looking for context for lessons. Trying to reach learners that are not mathematical. Fun and interest".

- Broadening students' perspective (15%). In this case the stress was put on the potential effects of this teaching as for example:
  Q2A) "Yes: Gives meaning to geometry. Helps students to develop visualization". Q2B) "Yes: Allow students to connect mathematics with other science. Allow work with real data. The tasks are mere formative -students need to read, understand the context, chose mathematics tools to use, decide the answers".
- Specific links between these two disciplines with mathematics, and to topics they related were mentioned. (15%)

Some participants specified the subjects they related to as for example:

> Q2A) "Yes: Geometry, landscape, paint". Q2B) "Yes: Biology, chemistry, physics".

Other participants stressed in both cases, the relationships that could be found between the disciplines and the context of teaching as for example:

> Q2A) "Yes. Some students are more interested in non-mathematical subjects (like art). The aesthetics of art and the aesthetics of mathematics are often related".
>
> Q2B) "Yes. Mathematics is often discovered as a way to explain science. I teach a course in mathematical modeling, and models from science are common examples".

Not including topics relating mathematics with art

35% of the participants stated that they did not include these topics in their teaching. Arguments were mainly based on students' or teacher's context, for example: "Not relevant for the students. No time". "It is not an essential aspect of my work. (It can be incidental)". "I'm a chemistry instructor at the university level".

Not including topics relating mathematics with science

20% stated that they do not include these topics in their teaching which is carried out at different levels. Arguments were presented in these responses referring to:

- Context (10%). Arguments refer to curricular demands or to institutional circumstances as for example: "Little call in my institution for *maths*. With science; some call for statistics with health sciences".

No interest is expressed regarding the inclusion of any topic of science.

- Lack of orientation and/or motivation (10%), as is shown in the response: "Haven't found the ways to for it in yet – But may do it this year. Others do this – I wanted to focus on something that's covered less".
  Another participant expressed the conviction of the relevance to include these topics: Q2b): "Yes. It is important to show *all* the relations between different topics".

Final remarks

Motivation and interest for students and/or teachers was found to be a major reason for including topics relating mathematics and sciences (45%) and/or topics relating mathematics and arts (30%). A response that might represent this emphasis on motivation is: "Because I like it, and what we teachers like helps us to teach better".

Some differences were found between the argumentations about the inclusion of mathematics and art topics and the inclusion of mathematics and science topics.

In reference to relating mathematics and arts some participants expressed: "Teachers and students find "lessons" involving art and mathematics engaging and motivating".

Other participants stressed an association to pleasant feelings for example: "It is colourful, fun, tactile, allows group interactions and opportunities for creative students to shine in a subject (mathematics) that they often don't".

These responses suggest that including topics of art might be associated with evoking pleasant feelings and to motivation.

In reference to relating mathematics and sciences some participants remarked other aspects: "Provides relevance to the topics". Also other participants expressed: "Helps maintain student interest better. Shows the interdependency of mathematics and other subjects (that mathematics as science cannot exist in isolation)".

These responses suggest that showing links of mathematics with science is perceived as a relevant argument. This argument was stated by the majority of participants (70%) for including topics of science, thus surpassing the motivation proportion (45%). This was not the case when considering the relations between mathematics and arts, where the corresponding proportions were 30% for motivation and 15% for showing links of mathematics with arts.

The context of teaching (stressing students' profile, curricular demands and institutional restrictions) seemed to be determinant for *not* including topics of other subjects in the teaching of mathematics.

Responses stressing "lack of material" and "discomfort with science" together with "fear of giving misleading information" should be considered as a call for attention with regard to future action of this TSG in order to play an assisting role for teachers. Against this background, the TSG organisers have decided to produce an independent booklet to account for the presentations in the TSG. The booklet is expected to serve as a reference on the new trends and developments in research or practice related to this topic.

In view of the growth of research in mathematics education over the last decades, it is remarkable that only very little attention has been paid to research on relations between mathematics and other subjects of art and science. Issues related to this topic are complex, because they comprise at least two different components, an extra-mathematical and a mathematical one. Further research is needed in this field. At one of the TSG sessions, a participants suggested the establishment of a network for senior researchers and graduate students involved in research on relations between mathematics and other subjects of art and science. Such a network might play an active role in information exchange and communication between mathematics educators interested in these relations and in the development of the theoretical framework that we are currently lacking.

This report has been prepared by Marta Anaya and Claus Michelsen. They are happy to be contacted at manaya@fi.uba.ar and claus.michelsen@dig.sdu.dk, respectively, for further information on the work of this TSG.