

TSG 25: Language and communication in the mathematics classroom

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Introduction

After a call for 12-page papers, all of the 29 manuscripts received were reviewed by three experts. On the basis of these reviews, three papers were chosen to be presented in joint sessions (one on the first and two on the last of the four days), 13 were allocated to three parallel sessions in subgroups (presented on the second and third days), and 12 were papers presented “by distribution”, published along with the presented papers on the TSG 25 web site.

The three plenary papers were based on three different aspects of language and communication considered to be of strong significance and general interest to the mathematics education community. All three reported on empirical research, underpinned by three different theoretical frameworks. Each paper was followed by considerable discussion and audience participation. The presenters and topics were as follows.

Session 1: Plenary presentation 1

Bill Barton and Pip Neville-Barton (New Zealand): “Undergraduate Mathematics Learning in English by Speakers of Other Languages”.

This opening plenary reported on three studies that investigated the dynamics of learning university mathematics taught in English, for students for whom English was an additional language (EAL). Two studies of first-year undergraduate students’ learning were followed by a third study of proof and mathematical argumentation used by third-year students whose native language was Mandarin. Contrary to common assumptions that mathematics is a subject in which language will have less impact on learning, in the first study EAL students experienced a 10% disadvantage in overall performance through lack of textual understanding. EAL students unjustifiably relied on symbolic modes to try to compensate for their textual disadvantage. However, the complexity of the issues was reinforced in the second and third studies, in which EAL students who had recently arrived in New Zealand self-reported levels of understanding similar to those of native speakers of English, on common types of problems.

Session 1: Plenary presentation 2

Morten Misfeldt (Denmark): “Computers as Media for Mathematical Writing: A Model for Semiotic Analysis”.

Using a semiotic framework to analyze the mathematical writing of research mathematicians and undergraduate students collaborating in groups, the presenter gave examples that focused on LaTeX and its role in the five functions identified for this mathematical writing, namely, (1) heuristic treatment, (2) control treatment, (3) information storage, (4) communication with peer collaborators, and (5) production of a paper. Analysis of examples by means of semiotic diagrams suggested a congruence



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Topic Study
Group 25



I C M E
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TSG

Topic Study
Group 25

between verbal mathematics and LaTeX code, while handwritten mathematics and standard mathematical code corresponded to the genre of writing in the *previewer* on the computer screen. The results offered insights into the use of different media at different stages of writing, and also into possible reasons why many mathematicians find LaTeX appealing.

Session 1: Plenary presentation 3

Norma Presmeg (USA): "Use of Personal Metaphors in the Learning of Mathematics"

The linguistic devices of metaphor (illumination of a concept or phenomenon by substituting a well known entity from a different domain for it) and metonymy (designation of a concept by means of one of its traits/attributes) have evoked interest in the mathematics education community in recent decades, but only a few studies have examined the roles of idiosyncratic personal metaphors in giving meaning to mathematical constructs. From an early study, examples of personal metaphors of secondary school students were used to illustrate source and target domains of metaphors, and their tension (elements common to both domains) and ground (differences). The role of associated imagery was discussed. A more detailed analysis was presented of later research on the personal metaphors of undergraduate students for *point*, *line*, and *plane*, in a university geometry course.

The presentations in three subgroups (numbered 1, 2 and 3) on the second day (1A, 2A, and 3A) and on the third day (1B, 2B, and 3B) are summarized as follows.

Subsession 1A:

Semiotic aspects of mathematics learning (chair and recorder: Carl Winsløw).

Silke Ruwisch (Germany) presented a paper entitled "Metaphors and metonymies and their impact in mathematical classroom discourses". She first defined metaphors and metonymies as general forms of reference (based, respectively, on *similarity* and *contiguity*). Then she discussed various uses of these concepts in the mathematics education literature and observed that in an educational perspective, *dynamic* aspects (how these forms of reference function and interact in learning and communication processes) seem to be mostly ignored at the expense of *static* aspects (particular occurrences in mathematical discourses). The subsequent discussion addressed, among other things, to what extent metonymies in the dynamic sense are linked to *abstraction*, such as when using letters to designate "arbitrary numbers" (which was an example of mathematical metonymy given by the speaker in her talk).

Hiro Ninomiya (Japan) combined two theoretical frameworks to analyse metacognitive forms of elementary students' note-taking in the context of working with decimal representation of numbers. The two frameworks were the Peircian theory of signs and nested chaining of signs, and Hirabayashi's notions of object representation, meta-representation and "other self". Ninomiya analysed and advocated the method of *reflective writing*, where the "inner self" (student interpretation of signification) is made explicit through metacognitive comments. In the discussion, some doubts were raised as to the appropriateness of identifying *object* and *representamen* directly with *questions* and *answers* in the analysed student work. Clarification was given as to the way in which the student writing had been brought about (namely, by students using a method exemplified by the teacher).

Related works by *Herbert Gerstberger* (Germany) (on cognitive transfer in understanding ratio) and *Filip Roubíček* (The Czech Republic) (on geometrization and semiotic representations) were presented by distribution.

Subsession 1B

Mathematics learning in an interactionist perspective (chair and recorder: Carl Winsløw).

Here, three connected papers were presented by researchers from J. W. Goethe University in Frankfurt/Main, Germany. The first paper, by *Götz Krummheuer*, outlined the research interests and theoretical basis for the work of the group. The author especially emphasised their interest in writing as a means for *externalisation* of student thinking, and for intensifying and enriching the participation of students in interaction about mathematics.

In this perspective, *Marei Fetzer* presented an empirical study using Toulmin's model of argumentation to analyse the structure of interactive problem solving, as observed in classroom interaction around certain mathematical writing tasks. Even students who are not attentive throughout such interactions – but who have access to their own writing pertaining to the problems – may participate meaningfully in specific parts of the oral argumentation process.

A more clinical type of study of students' mathematical writing was presented by *Christof Schreiber*. Using the software MS NetMeeting, the written and graphical interaction among pairs of 9-10 year old students around a simple word problem was monitored and recorded together with the oral utterances of each pair. Using Peirce's triadic sign model and Hoffmann's notion of "the general" (roughly speaking, elements of the context of signification), the data were analysed with a view to exhibiting the dynamic role of symbolic representations in communicating a solution from one pair to the other. The following discussion raised both technical and more general issues pertaining to the analysis of student writing and argumentation.

Related works by *Florenda Gallos* (The Philippines) (on students' private conversation) and *Cristina Tavares* and *Márcia Pinto* (Brazil) (on mathematics classroom discourse) were presented by distribution.

Subsession 2A

Bilingual learners of mathematics (chair and recorder: Linda Galligan).

The first presenter was *Linda Galligan*, (Australia) whose paper was entitled "The role of language-switching in bilingual students' processing of mathematics". This study investigated the language used by two Chinese-English bilingual beginning university students as they processed and solved various basic mathematics problems. The results showed uses of both Chinese and English at various levels of thinking. The results of the study aimed to begin to clarify the stages of interlanguage in mathematics for bilingual students, important for the teaching of mathematics to English Second Language (ESL) and Non-English Speaking Background (NESB) students at all levels of schooling.

The other presenter in the sub-session was *Lena Khisty*, (USA) (with Hector Morales). The title was "Discourse matters: Equity, access, and latin@s' learning mathematics". This paper discussed multilingual classrooms and the issues second language learners encounter in reform-based mathematics that emphasizes talking to learn. Highlighted was the interaction between academic language proficiency and mathema-



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Group 25



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Topic Study
Group 25

tics teaching and learning. Qualitative data from one primary and two secondary grade classrooms revealed the effects of this interaction. Results suggested that more attention needs to be paid to academic language proficiency development in the mathematics *context*. There were also two papers for distribution: *Serge Hazanov* (Switzerland), “Across the Language Border. Bilingual Mathematics for the International Baccalaureate” and *Soledad A. Ulep* (The Philippines), “Language practices in teaching and learning mathematics using English in a bilingual class in the Philippines”.

Subsession 2B

Classroom communication (chair and recorder: Linda Galligan).

Michaela Kaslova (The Czech Republic) gave the first paper entitled “Communication and interpretation of the solution – developing didactic thinking”. Students often know the answer but are not able to explain how to obtain it. The paper investigated the form of questions and solutions in open ended word problems – oral, phonetic, mathematical symbols, pictures, dramatizations; given to and by pupils of various ages and by adults (students and prospective teachers). Some elements of discussion raised by the analysis of primary and secondary pupils were given. The second paper was by *Rosa Ferreira* (Portugal) (with *Norma Presmeg*). The title was “Classroom questioning, listening, and responding: The teacher modes”. This study traced how two student teachers evolved in their teaching modes, that is, their interrelated questioning, listening, and responding approaches in the classroom. The relationships between the participants’ beliefs about mathematics teaching and learning and their dominant teaching modes were also investigated. This study raised several questions for future research and implications for teacher education. There were also two papers for distribution: *Minoru Ohtani* (Japan), “Symbolizing and Tool Use in Classroom Mathematical Activity: “Revoicing” as a Unit of Analysis”, and *Michelle L. Wallace* (USA) (with *Nerida F. Ellerton*), “Language Genres in school mathematics”.

Subsession 3A

Logic and language in mathematics discourse (chair and recorder: Viviane Durand-Guerrier).

The two presented papers shared the same theoretical framework, relying on first-order logic as an epistemological reference for analysing mathematical statements and reasoning in a didactic perspective, especially those involving quantification. *Viviane Durand-Guerrier* presented a paper entitled “Surreptitious changes in letters’ status in mathematical discourse”. After recalling three possible types of status of letters from a logical point of view, the author dealt in depth with a proof from a high school textbook in which there are numerous changes in the logical status of letters, without any indication of these changes. She then showed responses to a questionnaire attesting to students’ difficulties related to the logical status of letters in understanding mathematical statements. For a conclusion, the author stated that this phenomenon is widely underestimated and gave recommendations to take care of this in learning and teaching mathematics at all levels.

The paper by *Imed Ben Kilani* (Tunisia) was titled “Negation of universal statements between the demands of Arabic language, French language and mathematical logic”. The author presented the Tunisian context briefly, in which mathematics is taught first in Arabic and then in French. He then presented a logical and grammatical enquiry



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TSG

Topic Study
Group 25

showing that Arabic and French languages on the one hand, French language and logical-mathematical language on the other, are not congruent in Duval's sense. As a consequence, difficulties in handling negation are likely to appear in the Tunisian school context, especially when a change in language occurs. This was confirmed by the current research of the author. It is noteworthy that this paper suggests that there are broader issues in the case of bilingual classes.

Related papers by *Faiza Chellougui* (Tunisia) ("Articulation between logic, mathematics and language", closely related to the two presented papers), *Filippo Spagnolo* et al. (Italy) ("Logical-linguistic questions in European and Chinese cultures") and *Leigh Wood* (Australia) ("Language of university mathematics") were presented by distribution, complemented by a two minute presentation of each.

A brief but effective discussion followed, pointing to questions of rigor, effective impact on students of difficulties highlighted in theoretical research, and ways in which researchers might help teachers in addressing these difficulties.

Subsession 3B

Associated challenges in doing and formulating mathematics (chair and recorder: Viviane Durand-Guerrier).

This session was more epistemological and philosophical in focus. *Paul Ernest* (UK) presented a paper, "The semiotics of mathematical texts and myths". The author proposed a metaphor of the hero and creation myths as representations of human agency in following a proof and defining a mathematical theory, respectively. Through a dialogue between Logos and Mythos, the author assumed that, like all mathematical and scientific knowledge, mathematical proof in particular is a discursive form, even a narrative, thus it is amenable to the tools of linguistics, semiotics and literary analysis. This led the author, in particular, to consider the reading of a proof as a journey that could be modelled by a circle, due to 'the Cyclic Pattern of Mathematical Proof'.

One paper originally for distribution, by *David Wagner* (Canada), "Facing Mathematics: looking at and looking through mathematical symbols", was offered a ten-minute presentation. A paper by *Allan Tarp* (Denmark) entitled "Pastoral power in mathematics education: A postmodern sceptical fairy-tale study" was presented only by distribution.

During the discussion, the main focus was on how mathematics discourses carry validity and address the reader in order to convince him or her as well as contribute to a successful appropriation of mathematical knowledge.

All in all, the presented and distributed papers manifested the manifold extent of the theme of the TSG. The topics ranged from problems of bilingual learners, by way of interactions in mathematics classrooms and the relevance of metaphors and metonymies for mathematics learning and teaching, to logical aspects of mathematical texts and the use of different media when thinking and writing mathematically. Discussions revealed that the issues were of vivid interest to participants.

This paper was written by Norma Presmeg and Siegbert Schmidt. They will be happy to be contacted at npresmeg@msn.com and siegbert.schmidt@uni-koeln.de respectively, for further information on the work of this TSG.