

TSG 28: New trends in mathematics education as a discipline

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Introduction

The aims of TSG 28 were to portray some of the new trends in mathematics education research. This topic potentially includes many and diverse issues, far too many and too diverse to be treated in a single topic study group. The team has therefore decided to select two from among the potential issues:

1. Mathematics and cognitive science, with particular attention to theories of embodiment in mathematics education.
2. Combining quantitative and qualitative research methods in mathematics education.

This report will briefly summarize the proceedings for each issue separately and conclude with some speculation on connections between them.

The four working sessions were organized in the following way:

1. Opening session on topic 1 with two plenary presentations.
2. Opening session on topic 2 with two plenary presentations.
3. Parallel sessions on topic 1 and topic 2, with contributed papers and open discussion.
4. Concluding plenary presentations, one on topic 1 and one on topic 2.

In addition, brief discussions took place after each presentation. The presented papers and/or related papers by the authors are available from the website of the topic study group. (www.icme10.dk).

Topic 1: Mathematics and cognitive science

Three invited (plenary) papers and three contributed papers were presented. The invited speakers set themselves the task to stimulate the discussion on the main themes of embodied cognition, around questions such as "What are the bodily and biological mechanisms underpinning cognition?", and, particularly concerning mathematics, "What are the grounding metaphors used in the construction, systematisation and communication of mathematical thinking?"

The first plenary was given by *Nathalie Sinclair* (USA) on "Embodied and evolutionary perspectives in mathematics education". Nathalie posed the following questions: What is the role of bodily experience in our thinking? When we speak of theories of embodiment, what do we mean by that word, embodiment? Her presentation focused on these questions, using as a lens the interplay between embodied cognition and the use of dynamic geometry software. She identified three different ways of thinking about embodiment. The first relates to the way we embody different procedures, that is, learn to implement them in a way that is "automatic," without discursive mediation, that is, without thinking. Our body knows what to do, without conscious thought, or without computation. The second way lies in the origins of our ways of thinking: even the most



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abstract concepts have roots in our bodily experience. The third relates to the close coupling between motor and conceptual systems. On the one hand, when people conceptualise a category, they infer relevant actions that they could take on it; on the other, when people perform an action, it influences the construction of conceptual representations.

In the second plenary *Rafael Núñez* (USA) talked about “What embodiment for mathematics education? Issues and controversies from the perspective of cognitive science”. Rafael opened his presentation by giving a brief introduction to the role cognitive science has played in mathematics education. He then focused on embodied cognition and discussed the meaning of the concept of embodiment in the cognitive science of mathematics. He stressed the importance of studying the bodily-based inferential organization of mathematical concepts “in themselves”, and not only the bodily experiences that particular individuals may have in the process of learning. This can be achieved through ‘Mathematical Idea Analysis’, that is, the set of techniques for studying the inferential organization of concepts provided by implicit and largely unconscious cognitive mechanisms such as conceptual metaphors, conceptual blends, fictive motion, and gesture-speech production. Mathematics education could benefit from the study of these embodied mechanisms which play a crucial role in constituting the very fabric of mathematics.

Three contributed papers were presented. The first speaker was *Ornella Robutti* (Italy) on “The construction of mathematical knowledge through multiple perspectives”. The presentation was aimed at showing various applications of the basic metaphor of infinity in teaching and learning activities, some of which are mediated by the use of artefacts. Ornella highlighted the possibility that the artefacts help students in conceptualising mathematical infinity at a cognitive level, in the same way as metaphors do. The analysis suggested that in some cases even metaphors can be produced through the interaction with an artefact, and these kinds of metaphors can be introduced ad hoc, both by teachers and by students. The crucial point is not how metaphors could be used to do mathematics, but to approach the mathematical concepts at a cognitive level.

Francesca Ferrara (Italy) presented a paper on “Bodily experiments, metaphors, gestures and artefacts in grasping the meaning of a motion graph: a case study”. The presentation was aimed at analysing a learning activity, taking into account the contextual ingredients that shape the way 9th grade students interpret a graph arising on a symbolic-graphic calculator from a body motion in front of a sensor. The analysis reveals that a metaphor characterises the students’ cognitive behaviour in the process of understanding. However, this metaphor arises in a complex *context*, in which it alone cannot adequately explain the students’ cognitive processes. It is necessary to integrate the analysis with the consideration of the mediation role of the artefacts in use and of gestures that students need to represent and communicate ideas.

The third speaker, *Janete Bolite Frant* (Brazil), presented a study done with some collaborators (M. C. Barto, C. Dallanese, A. Mometti) on “Reclaiming visualization: when seeing does not imply looking”. This study was part of a larger study that investigates meaning productions for calculus contents by mathematics teachers and professors. Janete focused the presentation on the role of visualization in understanding calculus and in producing meaning for the concept of derivative of a function. Based on a neuroscience perspective according to which seeing does not imply looking, she produced



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an analysis of two episodes taking place in a computer laboratory used for a calculus class.

Finally, *Marianna Bosch* (Spain) presented a concluding plenary lecture on “Mathematical cognition and the anthropological approach to didactics: the institutional relativity of knowledge”. Marianna introduced the main tenets of the model of knowledge used by the anthropological approach. Then, she showed the main differences between the analysis provided, and the questions raised, by the embodied approach and those of the anthropological approach. She argued that the anthropological approach to mathematical cognition is a useful tool for mathematics educators to raise and solve educational problems. It is a theoretical tool that helps to better understand, but also a practical tool to progress in the teaching and learning of mathematics. Marianna concluded with a few short remarks about the role played by the body – and also by other kinds of material ‘artefacts’ – in the production and development of mathematical knowledge.

Topic 2: Quantitative and qualitative research

The issue

A judicious choice of research methodology depends on the aims of the research. Nevertheless, the potential benefits and drawbacks of qualitative, quantitative and mixed methodologies in educational research in general have recently become a common topic of discussion (see e.g., the November 2002 Theme Issue of the *Educational Researcher*). A common opinion expressed in this discussion is that quantitative research, when it is possible, yields results of greater validity than qualitative research but that because of the large influence of contextual factors and because of the ubiquity of interactions, quantitative research is often impossible in education. The time seems ripe for research in mathematics education to transcend the dichotomy of quantitative versus qualitative research and ask whether well designed combinations of quantitative and qualitative methodologies could yield results that would be more useful and more valid than those obtained from either type of methodology separately. The task set by the team for this part of the topic study group was thus to explore effective ways of combining quantitative and qualitative methods. Three invited (plenary) and two contributed papers were presented, each adding valuable experience and insight to the issues and questions under discussion.

The presentations

Kurt Reusser and *Barbara Vetter* (Switzerland) presented an invited paper entitled “Combining quantitative and qualitative analyses of lessons in (large scale) mathematics video studies. Insights from research and potential for teacher education”. They presented a study aimed at explaining outcomes of instruction in terms of a large number of variables related to teaching. The study is set against the background of a complex multilevel mediational framework of instructional quality and effectiveness. For example, the multiple levels of student, classroom, school and system were taken into account when collecting and analyzing data. The novelty of the study lies in the method of data collection, which combined video survey (tapes of 156 lessons) with the best of ethnographic case studies. A number of non-trivial methodological problems had to be solved; these included decisions on the level of classroom actions to be coded as well as training the coders. One of the advantages of a study like this is the option to zoom in from



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the survey data onto a single teacher or classroom or lesson for a detailed qualitative analysis.

*Jo Boaler*¹ (USA) presented an invited paper entitled “Studying a complex practice – using multiple methods to capture the relationships between teaching and learning”. She discussed methodological aspects of an intensive long-term study of three schools. Two of the schools had mainly traditional and a few reform classes; the third school had only reform classes. Low inference variables are relatively easy to measure using appropriate coding schemes; an example of low inference coding is whether teacher questioning is respondent to student actions. It turns out that it is more so in reform classes than in traditional classes but it also turns out that this difference does not explain the achievement gap between the reform and traditional classes. A deeper level of analysis concerns question types; for example, questions in traditional classes are almost exclusively factual whereas in reform classes they are varied. This difference had more explanatory power but it is more difficult to obtain quantitative data at such a deeper level. For even greater explanatory power, one needs interviews that are non-quantitative in nature: If teaching is to make meaning of how students think, then teaching cannot be measured quantitatively. On the other hand, the public domain and policy makers are being reached only by quantitative results. In summary, results at different levels need to be obtained and integrated.

Mi-Kyung Ju and *Oh Nam Kwon* (Korea) presented a paper entitled “Mixed methods: different ways of talking about students’ views about mathematics”. They made use of a mixed methods approach to evaluate the instructional design of an inquiry-oriented differential equations course in a university in South Korea. The questions steering their evaluation research required both quantitative and qualitative methods. They considered the development of reliable explanations as the strongest advantage of the mixed methods approach. Using their own research as an example, they showed that it is possible to increase this reliability by cross-checking whether explanations from different methods converge. They also pointed out, however, that their use of mixed methods led to a challenge, namely the development of divergent explanations of students’ views about mathematics. In retrospect, they looked at the challenge as a critical learning experience in that it provided an opportunity to witness the intricateness of the phenomenon under inquiry and to develop a richer description and explanation through crosschecking and reflection. They admonish researchers to be cautious in selecting measuring tools consistent with the theoretical perspective of their research.

Peter Petocz, *Anna Reid*, *Leigh Wood*, and *Geoff Smith* (Australia) presented a paper entitled “On becoming a mathematician: an international perspective for future professionals in the mathematical sciences”. They reported on their investigations of mathematics students’ ideas about working as professionals in the mathematical sciences, and on the impact that these ideas have on the students’ learning of mathematics. The research design used a three-phase combination of qualitative and quantitative approaches. Two phases of qualitative methods were carried out in order to carefully prepare a questionnaire that was then used to collect quantitative data. Thus the strengths of each methodology were exploited to increase the overall usefulness and researchers’ confidence in the results.

¹ Today, JB works in the UK.



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Jeremy Kilpatrick (USA) presented an invited paper entitled “Methods as ideologies: Is our research scientific or political?” About research in mathematics education, one can ask whether it is scientific and, if so, with regard to what science? One can also ask whether it is political and, if so, with regard to what politics? The history of research in our field shows that it began by emulating the natural sciences but over the past half century shifted rather drastically away from science even in the loosest sense. As that happened, and research became more qualitative, it got embroiled in controversies over “reform” and even found its way into partisan politics. All research involves comparison, whether explicit or implicit, and all data analysis is potentially quantitative. Begle’s call for an experimental science at ICME-1, however, is far from being met. Though he underestimated the role of value judgments in our research, he did help us see the value of empirical work. A merging of qualitative and quantitative methods – as shown, for example, in some studies of embodied cognition – can help move the field forward both scientifically and politically. The answer to the question posed in the title of this talk is that our research is both, scientific and political.

Discussion

The presenters showed a wide variety of ways to combine qualitative and quantitative research methods. They drew attention to the great advantages in terms of explanatory power that the combination of methods brings with it. They also pointed to the large amount of resources that are needed in order to carry out a quantitative analysis of qualitative data. Specific methodological difficulties, such as determining the level of inference at which coding is both feasible and relevant, were also mentioned. In summary, it appears that we can get significant and reliable results from qualitative research by up-scaling it, but that there are methodological problems to be considered and that the amount of resources in time and money associated with such up-scaling is very large.

This report was written by Tommy Dreyfus and Domingo Paola. They are happy to be contacted at tommyd@post.tau.ac.il and domingo.paola@tin.it, respectively, for further information on the work of this TSG.