

Discourse analysis and mathematics education: An anniversary of sorts

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Introduction

In 1979, twenty-five years ago almost to the day from the date of this lecture, Austin and Howson published their review article 'Language and mathematical education' in *Educational Studies in Mathematics* (10(3), 161-197). At the outset, they inform us that they started out to produce what they modestly referred to as an 'annotated bibliography', but the end-product was, in fact, an extended essay preceding a partially-annotated bibliography.

The task I have set myself to address in this talk is, in some sense, to attempt a similar overview, twenty-five years on. I have deliberately preserved Austin and Howson's choice of 'and' rather than the perhaps more familiar 'in' as the conjunction for my title, in order to emphasise two discrete, independently existing fields being laid side by side. However, I have wisely opted to impose some severe constraints, in order to render the task remotely manageable, as this area has grown enormously in the past quarter-century. [1]

Partly because of this explosion of texts, I have sneakily shrunk 'language' to 'discourse analysis'. This latter refers to an approach to certain elements of linguistic study which has really only come to any prominence in education within the same past quarter of a century, related to the emergence of *pragmatics* (as distinguishable from syntax and semantics) as a third core sub-area of general linguistic study. [2] Discourse analysis involves studying connected, naturally occurring speech or writing above the level of the sentence.

However, in this paper, I primarily propose to explore and exemplify the increasing application of concepts and methods of discourse analysis to problems of and enquiries into mathematics education (and into mathematics itself). My intent is to survey a fairly small field that is in the process of emerging, thereby both drawing greater attention to it and helping perhaps, in a small, mid-wifely way, further to bring it about.

Some features of discourse and their mathematical and mathematical education versions

I have divided the area I want to talk about into four categories and I shall say something here about the first three. [3] It should be understood that even within this localised arena of discourse analysis and mathematics education, any citations I offer are to be taken as exemplar rather than definitive and exhaustive, and refer exclusively to English-language literature. Needless to say, neither discourse analysis nor mathematics education are exclusively anglophone phenomena. Nevertheless, the increasing contemporary reliance on English as a medium of international academic exchange (whether in mathematics or mathematics education) means that the particular ways in which the phenomena of discourse analysis play out in English have a certain (unwanted?) significance over and above their specificities. Here are my four categories:

- (a) aspects of *voice*, reference (human and otherwise) and mathematical agency (Pickering, 1995; Boaler, 2002), including pronominal structure and shifts (Pimm, 1984, 1987, 2005; Rowland, 1999, 2000), deixis (Gerofsky, 1996; Netz, 1999; Rowland, 1992, 2000), nominalisation and verb mood (Morgan, 1996, 1998) and addressivity (e.g. Fauvel, 1988; Gerofsky, 1999; Phillips, 2002);
- (b) instances of *meta-discourse*, including propositional attitude and politeness (Bills, 1999, 2000), hedging (Rowland, 1995, 2000) and modality (Chapman, 2002);
- (c) components of *temporal structure*, including verb tense and sequential markers (Solomon and O'Neill, 1998; Pimm, 2005);
- (d) elements of *style*, including both mathematical and mathematics educational genres and specific genre features (Burton, 1996; Gerofsky, 2003; Love and Pimm, 1996; Marks and Mousley, 1990; Pimm and Wagner, 2003; Solomon and O'Neill, 1998).

Related to this laying out of an area is Balacheff's (1988) characterisation of written mathematical discourse as being 'decontextualised, depersonalised and detemporalised'. All three components of this triad of 'negations', these necessary (?) steps back from the immediate and the everyday, from the here-and-now, are of relevance here.

(a) *Voice*

In this sub-section, I will give examples concerning voice in written texts, comprising perhaps the less familiar sense of the term 'voice'. In a chapter in the *International Handbook of Mathematics Education*, Eric Love and I wrote:

"With any text always comes the question of voice. Who is (are) the author(s) and how do they acknowledge their presence in the writing? What pronoun(s) do they use to refer to themselves and the reader? [...] What is the relation of the author to the mathematical material? Is the reader addressed as an individual student? What evidence is there for the nature of a presumed 'ideal' reader on the part of the author as contrasted with any actual reader (e.g. what can I, as author, assume known)?" (Love and Pimm, 1996, p. 380)

One of the early constitutive texts of Western mathematics is Euclid's *Elements*. In a historical piece on different rhetorical styles on mathematics, John Fauvel (1988) contrasted that of Euclid with Descartes:

"Euclid's attitude [toward the reader] is perfectly straightforward: there is no sign that he notices the existence of readers at all. [...] The reader is never addressed." (p. 25)

It is interesting to me that an increasing number of historical/sociological studies concerned with mathematics and its history are drawing on a similar set of tools from linguistics as those undertaking mathematical education studies of school mathematical 'texts'. As a second instance, historian of mathematics Reviel Netz (1998, 1999), in his complex study of the archaeology of Greek geometric diagrams, has made use of Austin's (subsequently developed by John Searle) notion of speech acts (an early contributory element to pragmatics). Netz also draws on *deixis* as a tool, in order to

examine the subtle inter-relation of geometric diagram with text, and the use of letters in terms of indicating or specifying referents. Etymologically, ‘deixis’ relates to *deikneme*, proofs, an early Greek form based on a direct ‘showing’. I return to discuss Netz’s work further in sub-section (c) below, concerned with the presence and absence of time in mathematics.

Rowland (1992), in his work on the usefully ambiguous referent of the pronoun ‘it’ in discussing mathematical investigations, also refers to *deixis*, namely a frequently particularising use of language that ‘points’ to other elements in the surrounding context. He writes:

“I want, now, to return to the linguistic notion of *deixis*, which means the use of a word whose referent is determined by the context of its utterance. Deictic forms such as *you, now, here* are so commonplace that we automatically understand that their referents (respectively persons, times, places) are context-dependent variables.” (p. 46)

For some considerable time, my interest has been taken by pronouns in mathematical discourse. In a short article written back in 1984, I looked at the use of the pronoun ‘we’ in classroom speech and in textbooks, offering an account of this feature of mathematical speech and writing (one aspect of teacher/textbook voice), where it is deployed for different ends. Subsequently, Gerofsky (1999) has analysed the speech a university lecturer used when addressing fourth-year undergraduates, which she terms talking to ‘junior colleagues’, predominantly using personal, first-person pronouns: I, me, my. The academic used an intimate tone that she characterised as ‘broadly inclusive and egalitarian’. However, the same lecturer, when addressing first-year undergraduates, more often spoke in a persuasive tone (one she refers to as ‘a salesman’), highlighted by, among other features, the non-personal use of the pronoun ‘we’ (p. 46).

Further pronominal attention has been paid to ‘you’, as a marker of generalisation compared with ‘I’ by Rowland (1992, 1999). That change of pronouns is frequently accompanied by a verb tense shift, from that of the (particular) past tense of report into the (generalised, continuing) present tense of mathematical ‘foreverness’. Students, moving from reporting ‘what I did’ to giving an account of ‘what you do’, have often generalised along the way. They may well see the situation no longer as simply a matter of historical reporting of particular experience, but as involving a more general account of what *you* (i.e. anyone) would find (if you were to do it) or should do (in order to get it right).

As a further instance of the productive crossover between a study of professional mathematician’s textual practices and those of learners in schools, Morgan (1998), in her study of teacher’s evaluations of students write-ups of mathematical investigations, has articulated a number of features of the former. She carried out a detailed study of a published mathematical paper (taken from the *Journal of the London Mathematical Society*) in terms of its linguistic features. In it, Morgan found:

- *distant authorial voice* (e.g. use of passive, absence of direct author or reader references);
- *extensive use of nominalisation* (transferring processes and actions into objects), which allows mathematical objects to appear in subject position, suggesting they carry causality and agency, thereby obscuring human agency;

- *use of imperatives rather than pronouns* (e.g. directives to ‘let’, ‘consider’, ‘suppose’ and ‘define’), though this nonetheless indirectly suggests a human presence;
- *extensive use of connectives* (e.g. *hence, therefore, but*), explicitly marking relationships among sentences and clauses.

Despite a prevalent mathematical style seemingly set to deny this, Bakhtin (1952/-1986) has asserted that every utterance is always addressed to someone, whatever it may seem.

(b) Meta-discourse

There are elements of language whose primary purpose is to mark or otherwise indicate or pass comment on the author/speaker’s attitude towards another part of the utterance. One instance of this, the only one I will discuss here, comes with *hedging*, that is using specific expressions whose purpose is to soften the assertion with regard to its certainty or precision. (Linguist George Lakoff, 1972, p. 183, defines the function of hedges as a word or phrase “whose job is to make things fuzzier or less fuzzy”.) As a subject where clarity about issues of truth, as well as the scope and validity of claims, are central, the phenomenon of hedging in mathematics might be of special interest.

According to one of the early studies in this area (admittedly in the context of a paediatric physician’s talking with colleagues rather than mathematics – see Prince et al., 1982), hedges come in two basic sorts. These they term *approximators* and *shields*, according as whether the hedge, the imprecision marker, is introduced *within* the proposition itself or exists *outside* of the proposition, to comment upon it in the context of the speaker’s utterance. If P is some proposition, say the Poincaré conjecture or the Riemann hypothesis, then I can say ‘I think P is probably true’, which softens the more dogmatic assertion ‘P is true’ in two ways – first by saying I only ‘think’ it true rather than asserting it to be so, and secondly by deploying the additional hedge ‘probably’. Both of these ‘shield’ me from the risk of being told or shown that I was definitively wrong. (Indeed these two particular shields fall within a sub-class Prince et al. term ‘plausibility shields’.) Shields, then, are hedges which do not affect the nature of the proposition itself, which can remain as precisely and declaratively asserted as desired. Here it is my *commitment* to the proposition that is being hedged, not the statement of the proposition itself.

Then there are hedges which operate inside the proposition P. In order to illustrate these, I’ll mundanely take the proposition to be ‘two plus two is four’. I could assert ‘two plus two is *almost* four’, ‘or ‘two plus two is *approximately* four’ or even ‘two plus two is *round about* four’, all of which ‘hedge’ the proposition itself (i.e. interact with its statement itself rather than with revealing my level of investment in or commitment to its truth). These are all examples of a sub-class of ‘approximators’ that Prince et al. term ‘rounders’.

Mathematician Henri Lebesgue introduced a sophisticated and technical use of the approximator hedge ‘almost’, when he talked about properties holding ‘almost everywhere’, meaning that they were true apart from on a set of measure zero. And approximations can be made to a greater or lesser degree of arithmetical precision, which may allow them to be worked with mathematically (e.g. in error analysis).

(c) The question of time in mathematics and mathematics education

Mathematics is frequently described as ‘timeless’. (The most recent discussion of this by a professional mathematician I am aware of is by Mazur, 2004.) There are a num-

ber of systematic linguistic features that contribute to this sense of timelessness. One contributor is undoubtedly through the imposed syntax of mathematical prose itself, in particular by means of its customary verb-tense structure and other chronological markers. (Part of learning mathematics is learning to speak and, particularly, to write mathematically.)

As Morgan (1998) has noted, one significant feature of mathematical text is the extensive use of connectives, such as *hence*, *therefore* and *but*, which explicitly mark relationships among antecedent and subsequent sentences and clauses. Interestingly, most of the connectives used in mathematical English, such as *then*, *hence*, *since* and *when*, as well as the key terms *ever*, *always* and *never*, also have a chronological sense in everyday English.

The removal of the presence of mathematical author by genre conventions of either using ‘we’ or extensive passive constructions interacts with the perennial present tense. Additionally, anthropomorphising mathematical objects – so they are animated to have apparent agency of their own – contributes to their gaining a permanent, ‘timeless’ existence.

The most sophisticated discussion I have seen of the question of the essential ‘presentness’ of mathematical discourse is in a paper by Solomon and O’Neill (1998). Having looked at two examples of student work reported by Burton (1996), in terms of their temporal structure and markers, they then explore a historical example, from the writing of nineteenth-century mathematician William Rowan Hamilton. In particular, they look at broadly the same material presented in personal notebook entries, letters and more public academic papers. Solomon and O’Neill report the presence of contrasting structures in the writing:

“An examination of the letter and the notebook reveals a more complex structure than a simple narrative. The texts contain two distinct component texts: a *mathematical* text is embedded within a *personal narrative*. The difference between the texts is indicated in the tense system, the choice of deictic reference and the forms of textual cohesion employed.” (p. 216)

Solomon and O’Neill go on to observe that the personal narrative is written in the past tense and contains deictic time markers like ‘yesterday’, whereas the mathematical material is:

“[...] in the timeless present. At the same time there is also in the mathematical sub-text a distinct form of cohesion: the temporal order in the narrative gives way to a logical order in the mathematics. [...] mathematics cannot be narrative for it is structured around logical and not temporal relations.” (pp. 216-217)

Although this is not Solomon and O’Neill’s conclusion, quite simply put, I believe a major reason why mathematics comes to be seen as a-temporal is because that is how it is ‘supposed’ to be written about and how professionals do write about it. In school, and even in university, students learn that, “Last night I found that this happened” is historicised narrative, whereas shifting to “you find” or “one finds” and then to the stripped Euclidean assertion “this is always the case” brings rewards.

A second, quite different, way in which the human past is brought into the present in contemporary mathematics is by the widespread use of eponyms, namely naming concepts, results, theorems and principles by means of mathematicians’ names. According to Henwood and Rival (1979), Charles Darwin complained about this



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practice in biology more than a century ago on two grounds: inciting hasty, and therefore shallow, work (in order to ensure the patriarchal priority of surname), as well as the impenetrability of mere names over descriptive naming. Mathematical examples of the latter would be ‘Abelian’ rather than, say, ‘commutative’ groups and ‘Hilbert’ rather than, say, ‘complete complex normed vector space, whose norm is given by an inner product’.

I wish to add another ‘complaint’ here; the posthumous co-opting of the past by the present, in order to render mathematical styles apparently less temporal. By labelling a particular formulation as Lagrange’s or Cauchy’s theorem, for example, is to recruit their work into a modern idiom, into ‘our’ style of doing mathematics. I believe it is done, in part, to make them one of ‘us’.

In this way, the ‘timelessness’ of the notions, formulations and even styles of proof themselves become buttressed by the past and, thus, rendered invisible. What we lose (and are meant to lose) is a contingent and historical sense of our present-day mathematics. How can someone nowadays read Cayley or Jordan, having studied a course in group theory which included ‘Cayley’s theorem’ and the ‘Jordan-Hölder theorem’, and not assume that they were addressing the same objects, seen in the same axiomatically-specified way.

In the Ancient Greek world, there was considerable discussion of the ‘timelessness’ of geometry and the permanence of geometric knowledge versus the *genesis* (coming-into-being) of mathematical objects by means of constructions. Lachterman (1989) wrote:

“For Speusippus the language of *genesis* has an ‘as-if’ character. [...] we must not treat the constructions and motions on display in a geometrical proof as ‘makings’ in the course of actual performance, as time-consuming just for the reason that first this is done, then afterwards that is done, and so forth. [...] What, taken literally, seems now to be coming into being for the first time [...] must be regarded figuratively as having already been accomplished all along.” (p. 62)

Lachterman quoted Aristotle, in *On the Heavens*, arguing against a parallel between diagrammatic proofs and the cosmic myth put forward in Plato’s *Timaeus* [I 10 280a4ff]:

“They say that ordered things came to be out of disordered, but it is impossible for the same to be simultaneously disordered and ordered. There must be a genesis involving the separation {of things} in time as well. In the diagrams, nothing is separated in/by time.” (p. 63)

Lachterman concluded this section (p. 65) with an examination of the curious verb tense and mood of Euclidean construction language of ‘operations’ – the perfect passive imperative: examples include ‘Let such-and-such have been done’ and ‘Let it have come about that ...’.

“The perfect tense tells us that the relevant operation has already been executed prior to the reader’s encounter with the unfolding proof. [...] Euclid invites us, not to perform the operation on our own, nor to observe him performing the operation before our eyes, but rather to consider the operation as already anonymously [the agentless passive mood] performed

before the ‘present moment’ [...] This verbal operator does not so much suppress time as shift it backwards into an unnoticed past.”



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In mathematics, much is made of assertions by *fiat*, that is sentences imperatively beginning ‘Let ...’. But who is the suppressed person toward whom such an imperative is directed? At least with ‘Let us pray’ there is an implied audience or congregation. But are all ‘Let ...’ utterances implied inclusive first-person plurals? Or are they appeals for permission to a higher authority? Whence derives the authority to make such utterances?

In the English translation of Euclid, such fiats are almost entirely simply uttered, unattributed to any speaker: only occasionally will someone declare ‘I say that [...]’, followed by the specific statement of the result *in terms of the letters in the diagram*. The earlier *protasis* (“enunciation”) is usually simply asserted, without reference to any diagram or an asserter. (For an instance of this rigid structure for the setting out of a proposition, see Netz, 1999, pp. 10-11.)

In a paper entitled ‘Why did Greek mathematicians *publish* their analyses?’, Netz (2000) examines possible linguistic differences of voice and verb structure relating to texts recounting mathematical work carried out by the ancient and antithetical methods of analysis and synthesis. He remarks:

“Even without any second order pronouncements [i.e. meta-commentary about purpose or intention], there could have been suggestions of a sequence of discovery, e.g. using a past tense in the assertions of the analysis as opposed to the present tense in the assertions of the synthesis, or using a first person active for the constructions of the analysis as opposed to the third person passive for the constructions of the synthesis. But nothing like this happens, everything is in the present tense or the third person passive suggesting the impersonal work of mathematical necessity rather than the accident of authentic discovery.” (p. 146)

In many ways, it is precisely an exploration of this same question – albeit in the context of secondary school students’ reports of their mathematical investigations and the tacit judgements of worth in the examining teachers – that comprised the focus of much of Morgan’s (1998) study.

So the issue of time in mathematics is both complex and linguistically sophisticated. What would an elementary mathematics education instance of this phenomenon look like? In her dissertation on aspects of mathematical writing among nine-year-olds, Phillips (2002) reports of examples of arithmetical word problems some of her students created as part of their work on writing mathematical textbook chapters. Here are four fairly standard instances, showing a certain mastery of the genre conventions:

- There were 16 rulers. 7 of them were 30 centimeters long and 9 of them were 15 centimeters long. How many centimeters were there altogether?
- Sally and Patricia went out and ate sushi. The Salmon cost \$5.00 and the California rolls cost \$7.60. Sally and Patricia bought 6 Salmons and 4 California rolls. How much did they spend?
- The ceiling in the house is 3 meters high. Henry was 0.6 meters tall a year ago, and now the top of his head is 2.3 meters from the ceiling. How much has Henry grown in the last year?

- Sylvester spoke on the cellular phone from 5.45pm to 6.30pm. The phone was free after 6, but cost \$0.5 per minute before 6. How much did he have to pay.

For instance, the first problem is in the standard ‘mathematics problem’ voice: omniscient and non-personal. It is also completely in the past tense and the tacit framing of ‘if ..., then ...’ is observable. These authors have captured, perfectly, the ‘pretend this is so’ and ‘pretend this is important’ aspect of textbook word problems (as discussed in Shiu, 1988), as well as the ‘three-component structure’ referred to in Gerofsky (1996).

However, there were two interestingly anomalous instances in the set of fifteen. One feature to bear in mind is that in both of and only in these two problems below (out of the set of fifteen problems created) did the students use the names of real students in the class.

- Jane and Lucy both weigh 35 kilograms. Lucy went on a diet and now she is 30 kilograms and Jane has gained 7 kilograms. How much more does Jane weigh than Lucy?
- Clara has 32,002 strands of hair. Clara’s hair was all dark brown. Rebecca has 31,128 strands of hair. What was the difference between the number of strands of hair of Clara and Rebecca? And what information did you not need?

Looking at the verbs in the first of these problems, there is a complex and problematic switch of tenses (weigh, went, is, has gained, does weigh) from present to past to present to past to present and the presumed, created context of the problem makes this hard to rationalise.

Gerofsky (1996) draws on linguist Stephen Levinson’s (1983) distinction between L-tense (linguistic) and M-tense (meta-linguistic). In the context of word problems, L-tense is basically grammatical verb tense; M-tense refers to where there is some coding time (CT) event in the story itself and tense markers are determined relative to it.

“In an M-tense system, we distinguish the temporal location of events in relation to CT: *past* refers to events prior to CT, *present* to events spanning CT, *future* to events succeeding CT, *pluperfect* to events prior to *past* events (which are themselves prior to CT), and so on.” (p. 40; italics in original)

Gerofsky gives patterns of various L-tense sequences from mathematics textbook word problems (consistent L-past is one of them, as in the first problem of the conventional set given above), but none of them match the one being discussed here. She asserts:

“A great number of anomalies can be found which combine L-tenses in a self-contradictory way, that is, in a way that contradicts the usual use of L-tense in English, where the statements are assumed to have truth-value and the event is assumed to take place in a stable deictic relationship to coding time (CT) [...] I have found that determining M-tense in mathematical word problems is problematic.” (p. 40)



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Part of the difficulty of the first anomalous problem arises in relation to the L-present tense assertion in the opening sentence being taken to refer to the present, whereas the coding time in the problem actually seems to be cued by the use of 'now' in the second sentence, which would imply the first sentence is actually M-past and so 'should' have read "Jane and Lucy both *used to* weigh 35 kilograms". It is not surprising that these students have yet to master this sophisticated related and consistency of tense in a hypothetical setting (after all, Gerofsky gives a number of examples where adult textbook writers have failed to do so). But it also serves to reinforce how complex arithmetic word problems are as linguistic texts.

In relation to this issue about verb tense, Gerofsky makes considerable play of the fact that word problems have no truth value: the people and the events are fictional. Yet by using the names of real girls from the class in this problem, there may have been some interaction between the problem authors' real and fictional worlds. Whether this had anything to do with the tense confusion is hard to say. Nevertheless, it seems to me at least plausible to suggest that this might have played some interfering role in the creation, the mix of factual and counterfactual elements at work in the problems. Using pupil names, dates and class events or even specific events in the world dates a text or problem in an objective way. One of the things that these students complained about in their own textbook was unrealistic settings or out-of-date prices, saying it tripped them up. Arguably, it took them out of the mathematics text by breaking their suspension of disbelief.

I have chosen to spend time analysing arithmetic word problems here in part because linguistic analysis of word problems has been one of the earliest and consistent foci of attention of this sort of work over the past forty years. From early doctoral studies in terms of sentence length and specific grammatical or lexical features (mentioned in Austin and Howson's review), to the above accounts drawing on considerably more sophisticated pragmatic features and accounts, the humble but ubiquitous word problem has proven a worthy item of study.

In conclusion

In this brief account contained in this paper, I have attempted to portray some of the sorts of linguistic phenomena at work and at play within mathematics education and, indirectly, within mathematics itself. This sort of discourse analysis is really only in its nascent stages in our field and relies on linguistically sophisticated investigators within mathematics education. Possibly for too long, psychology has been unquestionably assumed to be the most key cognate social science for mathematics education. Perhaps, the time for linguistics has arrived.

Notes

[1] For even a miniscule sense of what has transpired under this general heading since then, and then listing only books in English, see Borasi and Siegel (2000), Brown (1997), Cocking and Mestre (1988), Dowling (1998), Durkin and Shire (1991), Ellerton and Clements (1991), Kieran, Forman and Sfard (2002), Lampert and Blunk (1998), Mousley and Marks (1991), Pimm (1987), Shuard and Rothery (1984) and Steinbring, Bartolini Bussi and Sierpiska (1998). (I have also left out mention here of books which I shall refer to specifically later in this paper, among many others.)

[2] Pragmatics can be specified as those aspects of language *use* crucial to understanding language as a system, relating meaning to speaker intention and context of use: central phenomena in pragmatics include deixis, hedging, implicature, presupposition, speech acts and conversational structure. For a core text on pragmatics in general, see



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Levinson (1983); for one related to mathematics education, see Rowland (2000); for resources on discourse analysis in the context of education, see Stubbs (1983) or Cazden (1988/2001).

[3] My recent thoughts about (d) can be found in Pimm and Wagner (2003), an essay review of Morgan (1998).

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