Understanding, assessing and developing young mathematical thinkers

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Abstract and introduction

This paper will outline a major research and professional development project involving 70 Victorian primary schools over three years. A major aim of the project was to identify processes for supporting and enhancing mathematics learning in the early years of school, particularly focusing on children of ages five to eight. The project involved four main components:

1. The development and refinement of a set of research-based "growth points" in Number, Measurement and Geometry, in order to map out the "typical mathematical landscape" of young children's developing knowledge and understanding of mathematics.

2. The development and use of a one-to-one, task-based assessment interview. This interview was used by project teachers with all of their students, at the beginning and end of the school year, and provided important information on what children knew and could do, and an excellent basis for lesson planning for individuals and the whole class. The interview was used on over 36,000 occasions during the project, providing rich data on children's understanding in the first five years of school.

3. A multi-level professional development program at state, cluster and school level for teachers, mathematics coordinators and principals. As well as input from the research team, the professional development program enabled teachers to share their learning with colleagues in a supportive, collaborative environment.

4. Case studies of six particularly effective mathematics teachers. These teachers were identified as those teachers whose children had shown the greatest growth in understanding (as indicated by the interview) in the first two years of the project. As a result, 25 characteristics of effective teachers of mathematics in the early years emerged.

In this paper, the characteristics and effects of each of these components will be discussed. Data are presented on growth in student understanding across the mathematical domains and grade levels. There will be a discussion of the professional growth of project teachers in general, and the practices of particularly effective teachers.

Background
The Early Numeracy Research Project (ENRP)[1] was established in 1999 by the (then) Victorian Department of Education, with a Prep(aration) to Grade 2 mathematics focus (the first three years of school in this state, with children's ages ranging from five to eight). The ENRP was a collaborative venture between Australian Catholic University, Monash University, the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. The project was funded for just over three years in 35 project ("trial") schools and 35 control ("reference") schools (for details, see Clarke, 2001; Clarke & Clarke, 2004).

The three main components of the early numeracy research project
The three main components of the ENRP are closely related to three terms used in the title of this paper. The framework of growth points provided a means for *understanding* young children's mathematical thinking in general, the interview provided a tool for *assessing* this thinking for particular individuals and groups, and the professional development program was geared towards *developing* further such thinking. In the following sections, each of these three components is discussed in turn.

It should be noted that, for brevity's sake, the term "understanding" is used in much of this paper. We are usually speaking of students' knowledge, skills and understandings, but even this, as argued by Kilpatrick, Swafford and Findell (2001), is somewhat limited in terms of an appropriate focus. Their preferred term "mathematical proficiency" includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Of course, these terms in turn need appropriate definitions, but all five aspects are of interest in the ENRP.

The ENRP learning and assessment framework
The impetus for the Early Numeracy Research Project was a desire to improve mathematics learning and so it was necessary to quantify such improvement. It would not have been adequate to describe, for example, the effectiveness of the professional development in terms of teachers' professional growth, or the children's engagement, or even to produce some success stories. It was decided to create a framework of key "growth points" in numeracy learning. Students' movement through these growth points in trial schools could then be compared to that of students in the reference schools.

The project team studied available research on key "stages" or "levels" in young children's mathematics learning (e.g., Bobis, 1996; Boulton-Lewis, 1996; Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; McIntosh, Bana, & Farrell, 1995; Mulligan & Mitchelmore, 1995, 1996; Owens & Gould, 1999; Pearn & Merrifield, 1992; Thomas, 1996, Wilson & Osborne, 1992; Wright, 1998; Young-Loveridge, 1997), as well as frameworks developed by other authors and groups to describe learning.

In developing the ENRP framework it was intended that the framework would

[1] The Early Numeracy Research Project is supported by grants from the Victorian Department of Employment, Education and Training (DEET), the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria. We are grateful to our co-researchers in ENRP trial schools and Pam Hammond (Senior Project Officer DEET), for insights that are reflected in this paper.

- reflect the findings of relevant research in mathematics education from Australia and overseas;
- emphasise important ideas in early mathematics understanding in a form and language readily understood and, in time, retained by teachers;
- reflect, where possible, the structure of mathematics;
- allow the description of the mathematical knowledge and understanding of individuals and groups;
- form the basis of planning and teaching;
- provide a basis for task construction for interviews, and the recording and coding process that would follow;
- allow the identification and description of improvement where it exists;
- enable a consideration of those students who may benefit from additional assistance; and
- have sufficient "ceiling" to describe the knowledge and understanding of all children in the first three years of school.

The decision was taken to focus upon the strands of *Number* (incorporating the domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies), *Measurement* (incorporating the domains of Length, Mass and Time), and *Space* (incorporating the domains of Properties of shape, and Visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There are typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children "count-all" to find the total ("1, 2, 3, . . . , 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. *Counting All* and *Counting On* are therefore two important growth points in children's developing understanding of Addition.

The six growth points for the domain of *Addition and subtraction strategies* are shown in Figure 1.

1. *Count-all (two collections)*
   Counts all to find the total of two collections.
2. *Count-on*
   Counts on from one number to find the total of two collections.
3. *Count-back/count-down-to/count-up-from*
   Given a subtraction situation, chooses appropriately from strategies including count-back, count-down-to and count-up-from.
4. *Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)*
   Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.
5. *Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)*
   Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.

6. Extending and applying addition and subtraction using basic, derived and intuitive strategies.
   Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

Figure 1. ENRP growth points for the domain of addition and subtraction strategies.

These growth points informed the creation of assessment items, and the recording, scoring and subsequent analysis, as is discussed in later sections.

In discussions with teachers, we came to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of conceptual landscape. However, we do not claim that all growth points are passed by every student along the way. For example, one of our growth points in *Addition and subtraction strategies* involves "count-back", "count-down-to" and "count-up-from" in subtraction situations, as appropriate. But there appears to be a number of children who view a subtraction situation (say, 12-9) as "what do I need to add to 9 to give 12?" and do not appear to use one of those three strategies in such contexts.

The interpretation of these growth points reflects the description by Owens and Gould (1999) in the *Count Me In Too* project: "the order is more or less the order in which strategies are likely to emerge and be used by children. . . . intuitive and incidental learning can influence these strategies in unexpected ways" (p. 4).

In discussing "higher" level growth points in a given domain, the comments of Clements, Swaminathan, Hannibal, and Sarama (1999) in a geometrical context are also helpful: "the adjective *higher* should be understood as a higher level of abstraction and generality, without implying either inherent superiority or the abandonment of lower levels as a consequence of the development of higher levels of thinking" (p. 208). Similarly, Konold, Khalil, Higgins, & Russell (2001), proposed five perspectives that children take in reasoning about data, and described them as follows:

> "These categories form a hierarchy of sorts, where a higher level subsumes or encapsulates lower ones. Different contexts may cue different views of data even within the same student. . . . Thus we see these not as levels or perspectives to graduate from, but rather to master". (p. 1)

Also, the growth points should not be regarded as necessarily discrete. As with Wright's (1998) framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (p. 702).

The ENRP task-based assessment interview

A major feature of the project is a one-to-one interview with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 40-minute period. The disadvantages of pen and paper tests have been well established by Clements and Ellerton (1995) and others, and these disadvantages are particularly evident with young children, where reading issues are of great significance. The face-to-face interview is an appropriate response to these concerns. The interviews are conducted by the regular classroom teacher in trial schools, and a trained team of interviewers in reference schools. A range of procedures has been developed to maximise consistency in the way in which the interview is administered across the 70 schools.

Although the full text of the ENRP interview involves around 60 tasks (with several sub-tasks in many cases), no child moves through all of these. The interview is of the form of a "choose your own adventure" story, in that the interviewer makes one of three decisions after each task, as instructed in the interview schedule. Given success with the task, the interviewer continues with the next task in the given mathematical domain as far as the child can go with success. Given difficulty with the task, the interviewer either abandons that section of the interview and moves on to the next domain or moves into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

All tasks were piloted with children of ages five to eight in non-project schools, in order to gain a sense of their clarity and their capacity to reveal a wide range of levels of understanding in children. This was followed by a process of refining tasks, further piloting and refinement, and where necessary, adjusting the framework.

The form and wording of the tasks are influenced by the growth points for which they are intended to provide evidence, while at the same time the consideration of the data provided by a given task can lead to a refining of the wording of a given growth point.

The interview provides information about those growth points achieved by a child in each of the nine domains. Figure 2 shows three questions from the interview, from the section on *Addition and subtraction strategies*. Words in italics are instructions to the interviewer; normal type are the words the interviewer uses with the child.

18) Counting On
a) Please get four green teddies for me.
Place 9 green teddies on the table.
b) I have nine green teddies here (show the child the nine teddies, and then screen the nine teddies with the ice-cream lid).
That's nine teddies hiding here and four teddies here (point to the groups).
c) Tell me how many teddies we have altogether. . . . Please explain how you worked it out.
d) (If unsuccessful, remove the lid). Please tell me how many there are altogether.

19) Counting Back
For this question you need to listen to a story.
a) Imagine you have 8 little biscuits in your play lunch and you eat 3.
How many do you have left? . . . How did you work that out?
If incorrect answer, ask part (b):
b) Could you use your fingers to help you to work it out? (it's fine to repeat the question, but no further prompts please).

20) Counting Down To / Counting Up From
I have 12 strawberries and I eat 9. How many are left? . . . Please explain.

Figure 2. An excerpt from the addition and subtraction interview questions.

For clarity, some instructions to the interviewer have been removed here. For example, lack of success with question 19 (in both parts *a* and *b*) would lead to the interviewer to skip question 20 and the remainder of the *Addition and subtraction strategies* section.

Question 18 provides information on whether the child is able to count-on or use a known fact, needs to count-all, or is unable to find the total by any means. Our aim in the interview is to gather information on the most powerful strategies that a child accesses in a particular domain. However, depending upon the context and the complexity of the numbers in a given task, a child (or an adult) may use a less powerful strategy than they actually possess, as the simpler strategy may do the job adequately in that situation. Questions 18-20 illustrate this well. Question 19 is often solved by children modelling the eight biscuits with their fingers and then counting back. By the nature of the numbers involved in Question 20, neither modelling the 12 objects nor counting back 9 is easy. Children are therefore given the opportunity to use a more sophisticated strategy (if they possess it), such as count-down-to 9 (11, 10, 9) or count-up-from 9 (10, 11, 12).

Wright (1998) highlights the challenge of determining the actual strategy used by a child in solving a problem, as “a child may unwittingly or intentionally describe a strategy different from the one used” (p. 703). Not surprisingly, teachers’ facility with determining the strategy used increases over time.

Two of our more interesting responses to the question, “how did you work that out?”, are “my brain told me” and “God told me”.

A professional development footnote at this point is that in making decisions about the strategies used by children in solving these problems, the teachers are themselves becoming increasingly comfortable with the distinction between the various strategies and their various levels of sophistication. This is an important step in being able to facilitate the movement of their children to higher level thinking during classroom teaching.

A teacher and coordinator in her third year of involvement in the project wrote “I found the data much more valuable this year as I have a greater understanding of the growth points and the direction that my children need to take”.

As well as moving carefully through the 20-page interview schedule, the interviewer completes a four-page Student Record Sheet. The information on this record sheet is then used by a trained team of coders together with a scoring algorithm to assign “achieved growth points” to each child for each domain. The rating of an individual child at a particular growth point is based on his or her responses to a number of different interview tasks. The raters demonstrated extremely high levels (all greater than 90%) of inter-rater reliability (Rowley & Horne, 2000).

It is important to stress that the growth points are big mathematical ideas or concepts, and that much learning takes place between them. As a result, a child may have learned several important ideas or skills *necessary* for moving towards the next growth point, but perhaps not of themselves *sufficient* to do so. Also, to achieve many of the growth points requires success on several tasks, not just one or some. This enables us to know that a child uses a more powerful strategy consistently and appropriately.

Of course, decisions on assigning particular growth points to children for the purpose of this research project are based on a *single* interview on a *single* day. A teacher’s knowledge of a child’s learning is informed by a wider range of information, including observations during everyday interactions in classrooms. However, teachers agree that the data from the interviews reveal student mathematical understanding and development, in a way that would not be possible without that special opportunity for one-to-one extended interaction.

Each year after the initial interview, teachers have been invited to comment on "highlights, surprises or patterns" that emerged from the interviews and the data. Common themes are given below, with an illustrative example in each case:

- *Surprise at what many children were able to do*
  "Some children did better than expected from my first impressions of them during the normal maths program. The one-to-one situation and wait time allowed them the opportunity to show what they knew".
  "Working with a gifted Prep who actually worked out the answers quicker than I did was a highlight. Reading 24,746,154 on the calculator. Amazing!"
- *Surprise at some difficulties children had*
  "A child of great potential, completed nearly all the interview, but couldn't tell the time".
  "To discover that some children who you thought had particular concepts couldn't use these/didn't have them—they were good at 'hiding' within the group".
- *The emergence of the quiet achievers*
  "In every class there is that quiet child you feel that you never really 'know'—the one that some days you're never really sure that you have spoken to. To interact one-to-one and really 'talk' to them showed great insight into what kind of child they are and how they think".
  "Quiet achievers (especially girls)".
- *The power of the interview data in informing teaching*
  "My greatest surprise was the wealth of assessment information gained from the assessment interview. . . . and how I've been able to adapt some of the ideas into my classroom practice".
  "The one-to-one contact enabled me to focus on what I have to work on to enrich their learning".
- *The level of enjoyment and confidence displayed by the children during the interview*
  "The greatest highlight was that no matter at what level the children were operating mathematically, all children displayed a huge amount of confidence in what they were doing. They absolutely relished the individual time they had with you; the personal feel, and the chance to have you to themselves. They loved to show what they can do".

Student performance data from the first two years of the project will be presented later in this paper.

The professional development program

The professional development program occurs (formally) at three levels. The 250 or so teachers from trial schools met with the research team each year for around five full days, spread across the year. The focus of these days was on understanding the framework and interview, and on appropriate classroom strategies, content and activities for meeting identified needs of their students. Many teachers commented that their own mathematical knowledge has been enhanced considerably as they have focused on children’s mathematical thinking. Readings were provided, as were follow-up tasks, for later sharing. In conjunction with these meetings, Early Numeracy Coordinators from the trial schools met for an additional three days each year, and the principals for two days. The focus of these days was on finer grained data analysis and the development of school leadership roles within the ENRP.

On four or five occasions each year, the teachers met in regional cluster groups for two hours, usually after school. Each cluster contains from three to five school teams. One member of the university research team was responsible for each cluster group. The focus of these meetings is to complement the statewide professional development. There was usually a time of sharing, during which teachers discuss readings or particular activities or approaches that they have tried since last meeting together. This was followed by the content focus for the day, and further tasks are set that need to be completed before the groups meet again.

The third level of professional development took place at the school and classroom level. The cluster coordinator visited each school approximately three times per year, spending time in classrooms team teaching or observing, participating in planning meetings, jointly leading parent evenings, and acting as a “sounding board” for teachers, coordinators and principals. In addition, the Early Numeracy Coordinator at each school conducts weekly or fortnightly meetings of the "professional learning team”, to maintain continuity, communication, team cohesion and purpose.

The student data

As was described earlier, all children in trial schools and a sample of approximately 40 children in each reference school were interviewed early in the school year (in a three-week period in February/March) and late in the school year (in a three-week period in November), using the ENRP task-based interview.

In the discussion below, the association between the growth points and the interview tasks must be kept in mind. As already mentioned, achievement of a particular growth point in the context of the following discussion means the capacity to successfully answer a series of particular questions on an interview, on a particular day. Changes in the interview arrangements or even subtle changes to questions are likely to yield different results.

Some data from the domain of addition and subtraction strategies

The ENRP has provided a unique opportunity to gather data on what large numbers of young children know and can do in various mathematical domains. Over the course of the project, approximately 36,000 interviews took place.

The trial schools were chosen to represent a diversity of school sizes, geographical locations, socio-economic levels, and English-speaking backgrounds across Victoria. The reference schools were chosen to carefully match the trial schools on all these variables. Given that the teachers in those schools have been “uncontaminated” by contact with the research team and the professional development program, the children in their classes provide a useful measure of what “typical children” can do. In order to give an accurate picture of typical children therefore, in much of the following discussion, the data used will be from reference schools.

Addition and Subtraction is the domain for which data are provided in Table 1. For particular grade levels, the percentage of children achieving each growth point (or better) is given. Data are given for children commencing school, and then at the end of the first, second, third, fourth and fifth years of school.

As an additional minor project, a stratified random sample of Grade 3 and 4 children was interviewed in November 2000, to explore the usefulness of the interview and framework for describing the understanding of eight- and nine-year olds. The exercise was most useful, however it needs to be stressed that the ENRP interview was created for use with P-2 children, and there are key content domains appropriate for older students not assessed (e.g., fractions) that would need to be given attention if the interview was extended to Grade 4.

	Prep (Feb) *n*=506	**Prep (Nov) *n*=506**	**Gr 1 (Nov) *n*=488**	**Gr 2 (Nov) *n*=446**	**Gr 3 (Nov) *n*=187**	**Gr 4 (Nov) *n*=172**
1. Count all	44	80	94	99	99	98
2. Count on	6	28	69	88	96	96
3. Count back/ count down to/ count up from	0.2	4	22	55	86	94
4. Basic strategies	0	1	10	38	77	92
5. Derived strategies	0	0	3	10	35	51
6. Extending and applying	0	0	0	0.6	3	13

Table 1. Percentage of reference school children in 2000 at each addition and subtraction growth point or above, by grade level (%)

The table shows the progress that young children make over time in using increasingly sophisticated strategies in addition and subtraction situations. A rough summary is that most children are able to count-all by the end of the first year of school (80%) and develop counting on by the end of Grade 1 (69%). By the end of Grade 4, just over half of the students are able to use both basic and derived strategies (51%).

One of the more interesting aspects of these data is the percentage of children who successfully used what we have termed basic and derived strategies in addition and subtraction situations. These include the use of commutativity, doubles, near doubles, combinations that add to 10, adding 10, and fact families, as well as the use of known facts. Given that only 10% of children at the end of Grade 2 have proficient use of these strategies, it adds further weight to the argument (see, e.g., Kamii & Dominick, 1998, Narode, Board, & Davenport, 1993; Plunkett, 1979) that teaching children two column addition and subtraction *written* algorithms in the early years of school is inappropriate.

One of the underpinning ideas of the framework, as stated earlier, was that it would "have sufficient 'ceiling' to describe the knowledge and understanding of all children in the first three years of school". It was hoped that all children would be challenged by the interview tasks across the various domains. In 2000, after around 9000 children had been interviewed (including 564 children at Grades 3 and 4), a Grade 2 boy became the first (and only to this point) student to successfully complete all interview tasks. The aim of challenging all students would appear therefore to have been achieved.

Teachers' stated professional growth

One purpose of the ENRP framework is to provide a means of quantifying young children's numeracy learning. However, we are at least as interested in identifying factors that may contribute to such learning. To complement the data on children's learning, a range of other data are being collected, including detailed questionnaires on teachers' beliefs and understandings about numeracy learning, regular journals kept by Early Numeracy Coordinators (the leaders of the professional learning teams in each school), as well as teacher and principal data on the effect of the project on teaching practice, student attitudes to mathematics, and home/school community links.

Given the clearly successful efforts of trial school teachers in developing children's mathematical skills and understandings in 1999 and 2000, it becomes increasingly important for the research project to study successful teachers' practice to try to discern those aspects of "what the teacher does" that make a difference. At a statewide professional development day in 2000, teachers were asked to identify changes in their teaching practice (if any). There were several common themes:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on "pulling it together" at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and "real life mathematics"; and
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Several of these themes are evident in the following quote from a teacher:

> "The assessment interview has given focus to my teaching. Constantly at the back of my mind I have the growth points there and I have a clear idea of where I'm heading and can match activities to the needs of the children. But I also try to make it challenging enough to make them stretch."

As Barbara Clarke (2000) noted, "teachers in the Early Numeracy Research Project have a clearer picture of the typical trajectories of student learning and can recognise landmarks of understanding in individuals" (p. 13).

In 2000, teachers were also asked to comment on aspects of children's growth that they had observed which were not necessarily reflected in movement through the

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growth points. Although the research team has a great interest in cognitive growth as demonstrated by the response to interview tasks, growth can take other forms (e. g., productive disposition, as identified by Kilpatrick, Swafford, & Findell, 2001). It is important to document these other forms of growth.

Common themes were the following:

- children are better at explaining their reasoning and strategies;
- children enjoy maths more, look forward to maths time, and expect to be challenged;
- the development of a "give it a go" mentality is evident, with greater overall persistence;
- children are thinking more about what they have learned and are learning; and
- all children are experiencing a level of success.

One teacher commented on her children's positive attitudes to mathematics:

> "Children seem to be more enthusiastic, take more risks and have more confidence in their abilities. They can't wait to participate. They're excited about maths. For example, we brainstormed the combination of green or red lollies to make 10 and when the children opened their bag, they exploded with excitement! "I've got 3 and 7!" "I've got 2 and 8!" All this over adding to 10!!"

Research as a powerful professional development trategy

Clarke (1994) argued that the research literature provides key principles for effective professional development. These are that professional development should:

1. Address issues of concern and interest, largely (but not exclusively) identified by the teachers themselves, and involve a degree of choice for participants.
2. Involve groups of teachers rather than individuals from a number of schools, and enlist the support of the school and district administration, students, parents, and the broader school community.
3. Recognise and address the many impediments to teachers' growth at the individual, school and district level.
4. Use teachers as participants in classroom activities or students in real situations, modelling desired classroom approaches during in-service sessions to project a clearer vision of the proposed changes.
5. Solicit teachers' conscious commitment to participate actively in the professional development sessions and to undertake required readings and classroom tasks, appropriately adapted for their own classroom.
6. Recognise that changes in teachers' beliefs about teaching and learning are derived largely from classroom practice; as a result, such changes will follow the opportunity to validate, through observing positive student learning, information supplied by professional development programs.

7. Allow time and opportunities for planning, reflection and feedback in order to report successes and failures to the group, to share “the wisdom of practice”, and to discuss problems and solutions regarding individual students and new teaching approaches.
8. Enable participating teachers to gain a substantial degree of ownership by their involvement in decision making and by being regarded as true partners in the change process.
9. Recognise that change is a gradual, difficult, and often painful process, and afford opportunities for ongoing support from peers and critical friends.
10. Encourage participants to set further goals for their professional growth. (p. 38)

In considering the various features of the ENRP, we believe that all these key principles have been met. Although the project teachers were not part of the design process originally, every endeavour has been made to accommodate their input along the way.

During the last ten years, there has been much emphasis given to the power of the “assessment tail in wagging the instruction dog”. One argument is that if high stakes assessment requires certain forms of performance, this is likely to result in teachers preparing students for this assessment, by aligning instruction with assessment. Clarke, Stephens, and Wallbridge (1993) referred to this as the “ripple effect”. Depending upon the context and your point of view, this can be a good or a bad thing.

In the case of the ENRP, the requirement of teachers to participate in the assessment interviews has meant that they have been involved deeply in researching the understanding of their children, as individuals and as a group. Having access to data from a much larger group of students has also enabled them to consider patterns or trends and to start to consider reasons for these. Ongoing assessment and interviews in the latter part of each year have provided an opportunity to evaluate the effectiveness of their teaching across different domains. This process has proved very powerful in teachers’ own professional development. They have increased their knowledge of how children learn mathematics in general, they have a much clearer picture of their own children’s understanding, and they have a repertoire of teaching approaches to enhance this understanding. The role of the co-researcher has therefore been a powerful professional development tool.

The research team has noted with considerable pleasure, particularly in the third year of the project, the increasing fluency of trial school teachers with mathematics education research terminology, and the willingness to engage in complex ideas over extended periods. It appears that the “shared language” about young children’s learning, so evident among teachers in the context of literacy, is becoming a reality in mathematics as well.

The framework as a lens for teachers

When the ENRP learning and assessment framework was first developed, a major purpose for its creation was to enable a measure of the effectiveness of the professional development aspect of the project, by monitoring student movement through the growth points. However, the framework has proved powerful in a variety of other ways.

Teachers are increasingly "owning" the framework, and using it to enhance their own understanding of children's mathematical learning. Teachers' understanding of the framework is enhanced by their familiarity with the interview. As the framework becomes better known, teachers view student responses during the interview in the light of their understanding of the growth points. Most importantly, the growth points provide a kind of "lens" through which children's mathematical thinking can be viewed, in all individual, small group and whole class interactions.

In summary, the Early Numeracy Research Project offered:

- A research-based, readily understood framework for understanding children's mathematical thinking.
- A powerful, one-to-one interview to gain a picture of individual and group understanding of big mathematical ideas.
- A professional development program and approach designed to support teachers to build upon what children know and can do.

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