# Future teachers use technology to explore concept development in mathematics

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***Abstract***

*This paper discusses the evolution of mathematics programs at Brock University, outlines the discipline requirements to be a teacher in Ontario, and provides examples of mathematics course and summer project work completed by future teachers. Features of all the examples are that they focus on mathematics concept development, and they make extensive use of computer technology.*

*The education of future teachers in Ontario is, for the most part, based on a consecutive model. That is, students complete an undergraduate degree before they apply to a Faculty of Education for a one-year pre-service program. At Brock University we have negotiated a number of concurrent mathematics and education programs in which courses and experiences for the undergraduate and education degrees are done concurrently.*

*In the early 1980s, some members of the Mathematics Department started to integrate technology into their courses in a substantial way. Since then, others have taken up the cause, and now the Department has instituted its core mathematics program MICA (Mathematics Integrating Computers and Applications). The pedagogical goal of the MICA program is to help students internalize a unified framework of mathematical concepts by interpreting them computationally, visually and algebraically. Therefore future teachers at Brock University develop their mathematical knowledge in an environment that nurtures concept development and that values the use of technology.*

## Ontario's environment for the education of future mathematics teachers

In Canada education is a provincial responsibility and in Ontario teacher education follows a consecutive model, where future teachers first complete a university degree and then apply to a Faculty of Education. The normal pattern is a three or four year undergraduate degree followed by one year in a Faculty of Education after which one is certified to teach in the Province of Ontario. Admission into Faculties of Education is based on a number of criteria including marks achieved in the undergraduate program, a portfolio, and undergraduate discipline requirements. For most programs in these Faculties there are far more applicants than positions and students must present an average of at least 75% in their undergraduate program. Their portfolio outlines experiences and responsibilities with children, in schools, in camps, in tutoring situations, etc., and this can account for as much as 40% of the admission criteria. Undergraduate discipline requirements depend on the school level certification. In this paper we summarize and simplify these requirements into elementary, middle, and high school certification. There are no subject specific requirements for elementary school certification and teachers are home-room teachers responsible for most disciplines. A minimum of three courses[1] in one subject taken from a list of 'teachable' subjects is required at the middle school level. These 'teachable' subjects include those that one would normally expect. Teachers at this level also teach across most disciplines. At the high school level candidates must present a minimum of six courses in one of a list of 'teachable' subjects and at least three courses taken from another subject from that list. Normally 'specialists' present an Honours Degree in their primary subject.

Because little time is allocated to subject specific courses in the BEd degree, the consecutive teacher education model carries with it a number of implications for university mathematics departments and for groups interested in mathematics education. At the elementary level, mathematics departments need to be pro-active and offer specially designed mathematics courses. At present the great majority of elementary teachers enter their teaching career with no postsecondary mathematics education. They therefore have very little understanding of mathematics, and how to teach it as a living discipline. Mathematics departments should be even more concerned about the mathematics education of teachers at the middle school level. Unfortunately very little has been done. In Ontario, middle school students begin their transition from arithmetic to algebra, in geometry they move from the visual/observational to the descriptive/analytical/relational, and they start their experiences in probability and data analysis. Middle school teachers need understanding of mathematics beyond an ability to perform a set of algorithms. At first sight undergraduate mathematics programs for secondary school teachers appear to be less problematic. But are they? Are mathematics teachers taking appropriate mathematics courses for their future career? Are they getting a breadth of experience in mathematics? What about future teachers who have a major in another discipline and have three courses in mathematics? Are these future teachers selecting courses that provide a breadth of experience in mathematics and that present mathematics as a living discipline? Or is their mathematics a compendium of techniques? Do they understand what mathematics is and what mathematicians do?

Summarizing, in Ontario the great majority of elementary school teachers have no post secondary school mathematics education; there has always been a shortage of middle school teachers who have any undergraduate background in mathematics; and about fifty percent of high school teachers teach mathematics with mathematics as their second teachable subject. Recent data shows that Ontario is now also experiencing a shortage of mathematics teachers at the high school level. The Mathematics Education Forum of the Fields Institute for Research in the Mathematical Sciences (web reference 1) has undertaken an initiative to address this concern by setting up a web site (web reference 2) that aims to interest mathematics students in the teaching profession and also aims to encourage future teachers to increase their level of mathematics education.

Jeremy Kilpatrick (2003, pp. 320,321) points to an evolution in school mathematics curricula in North America during the second part of the 20th century. For the first part of the century there was a clear division between primary and secondary mathematics education, at the primary level the curriculum emphasized the practical while at the secondary level the curriculum emphasised the intellectual. In the second part of the century and with the growth of education for the few to all, the mathematics curriculum has adapted by moving some of the intellectual into the earlier grades. This evolution has serious implications for the mathematics education of elementary and secondary school teachers. Elementary school teachers now have to have an

[1] A course in this context is a full year course.

understanding of generalization, bridging between different representations, etc. while secondary school teachers must now have experience in applications and modelling. The present Ontario curriculum also requires teachers to be comfortable teaching with technology and to reflect how students learn mathematics in a technological environment. As this paper will explain future teachers who study mathematics at Brock University are particularly well prepared to use technology in a reflective way.

## Brock's mathematics education environments

### *The core mathematics program*

Although mathematical knowledge has expanded in the last fifty years the traditional mathematics undergraduate degree has changed little, more options have been introduced in the upper years but the first two or three years are not very different from the ones I completed some forty five years ago. Lynn Steen (2003, pp. 193,194), in his article "Analysis 2000: Challenges and Opportunities" reflects on the role of analysis in the undergraduate mathematics major at the end of the 20th century. Let me quote one of his paragraphs.

> "During the last half-century mathematics has expanded enormously, both in the diversity of its specialties and in the pervasiveness of their roles in society. These so-called mathematical sciences encompass a diverse and rapidly expanding part of human intellectual accomplishment. Although still vigorous both in its own right and as a supporting tool for other parts of mathematics, analysis now represents a much smaller fraction of mathematical practice than it did fifty years ago, before linear programming or bioinformatics, before data mining and string theory."

He continues

> "Paradoxically, during this same half-century the role played by calculus in education, has expanded enormously, often irrationally. Today the mathematical focus of secondary school is calculus, not mathematics. Even as the mathematical sciences have built avenues of intellectual exchange extending in many different directions, the signposts of society still direct everyone to enter mathematics on the traditional highway of calculus. The question that we must confront at the start of this new century is whether this traditional route still serves mathematics well or whether it might not be prudent to explore other options through which students can enter the world of mathematics."

When I arrived in St. Catharines in 1967, Brock University had not erected its first building and had a student population of 300 students. As members of a fledgling Mathematics Department we were fortunate to have the opportunity to develop a mathematics programs from scratch. After thirty years the university has expanded, it has a population of over 16,000 students, and over half of the original members of the Mathematics Department have retired. They have been replaced by a new cohort of young energetic mathematicians. This was an opportune time to completely review Brock's mathematics major program, and to structure a core program for the mathematics students that Brock is now attracting, a program that would prepare these students for the 21st century. A committee was charged to first develop scientific and pedagogical goals, and to follow that by guiding principles. Details of the approved core mathematics program can be found in the Brock Teaching Journal (web reference 3 p 1), and its guiding principles are:

1) Encourage creativity and intellectual independence.
2) Develop mathematical concepts hand in hand with computers and applications.
3) Guarantee prerequisites.
4) Strengthen the Concurrent Education programs.
5) Create upward mobility.
6) Strengthen ties with other departments.

The result of this initiative is a core mathematics program for all majors and co-majors that integrates computers, applications and modelling, and that creates upward mobility for students with different educational goals. In my view the new faculty, the new majority, needed to commit to their own program. In describing this initiative, my point is not to suggest some universal undergraduate mathematics major program but rather to propose that all departments of mathematics need to review their programs in light of the developments in mathematical knowledge, in mathematics applications and modelling, and in the power of evolving technologies. Departments of mathematics also need to consider the goals of the students they educate, and they should seriously examine whether their programs are preparing their students for the 21st century.

***Concurrent education for future teachers***

One of the guiding principles listed above is ‘Strengthen the Concurrent Education programs’. These programs have been a priority for the Department of Mathematics for more than fifteen years.

Although mathematics educators have repeatedly expressed their concerns that there are so few middle school teachers with any significant education in mathematics they have done little to remedy the situation. Teacher education involves and impacts many different interest groups and Province wide changes are difficult to achieve. In an effort to have some impact on the situation, the Department of Mathematics launched a Concurrent Program in collaboration with members of the Faculty of Education and other members of the Faculty of Mathematics and Science. What is a concurrent education program? It is a program that integrates the undergraduate and education degrees. Many different versions of these exist around the world but they are usually centered in a Faculty of Education. In Brock’s case it requires the collaboration between two or more faculties that respects different points of view that negotiate course offerings, sequencing, etc. with an open mind and for the best interest of the students’ future career in teaching. In a consecutive program students would complete their undergraduate degree before doing their education degree and teacher qualification. In a concurrent program they do their undergraduate degree while also taking some of their education program. The latter engages students in issues of pedagogy starting in their first year.

A concurrent program is an attractive choice for students because, while there is a shortage of places in Ontario Faculties of Education, the program guarantees students a place in Brock’s Faculty of Education, provided they continue to meet certain conditions involving marks, course selection, and so on. The first program launched was highly structured and demanding in its diversity of emphases of mathematics, science and education. Professors report that concurrent education students form a real identifiable community, not only because they know each other and take most of their courses together, but also because they are proud to be in the program. Members of faculty enjoy the dynamics that these students generate in their mathematics classes. They are eager to share their knowledge and are ready to ask questions. The first program consisted of the following required (full year) courses; six in mathematics; three in different science; a number in education; one in child and youth studies; one in psychology; and one selected from the humanities. The program aims to provide a breadth of experience while it retains a concentration in mathematics. Students must maintain a 75% overall average. In mathematics the students are exposed to different areas of mathematics, that include calculus, linear algebra, discrete mathematics, combinatorics, probability, statistics, geometry, applied abstract algebra, geometry, history of mathematics and teaching/learning mathematics at the middle school level.

Recently other concurrent education programs have been launched at Brock. In these new programs students have more flexibility in their choice of a second specialization but where their program is still just as demanding in their requirements of various subjects that prepare them for a future in teaching and education. The Mathematics Department's focus on concurrent education has produced benefits for future teachers not in these programs.  One of the objectives in the restructuring of the core mathematics program was to provide upward mobility. To do this the Department critically examined the prerequisite knowledge for each course. By making relatively small changes in course content and approach students not majoring in mathematics are now able to access courses that were previously restricted to mathematics majors or co-majors. As a service to future elementary school teachers, a course was developed for students who had little previous success in mathematics. It is very rewarding to see future teachers overcome their phobia of mathematics and to build their confidence in the subject.

***Building a technological learning environment in mathematics education***

In the early 1980s, the author started integrating technology in undergraduate mathematics education at Brock University. With a colleague from the Université Laval, Bernard Hodgson, we wrote a chapter for a 1992 UNESCO report (Hodgson and Muller, 1992) entitled "The influence of computers and informatics on mathematics and its teaching". In it we suggest that transportation systems are an analogy for the evolving mathematics computer systems and their implications in undergraduate mathematics. Each of the bicycle, car, bus, train, and airplane provide different opportunities and meet different needs for travel. Selecting the appropriate modes of transport to go from A to B that meet the time, financial, entertainment, etc. constraints, does parallel the decision to use the appropriate technology in undergraduate mathematics to meet pedagogical, mathematical, and student learning and career goals. Just as it is now unconscionable not to educate the traveller about various means of transport, so it is not to educate undergraduate students in mathematics courses about the use of appropriate technology. Universities have a responsibility to inform and educate their students about the latest technologies and to allow them to choose what is most appropriate for their education, goals and development. In mathematics education at the undergraduate level the following question should be addressed: what would sequencing, within mathematics courses and within mathematics programs, look like if it was based on a conceptual hierarchy, while allowing students to use technology to meet their needs in the technical hierarchy? Let me expand a little on the difference that I see between a conceptual and technical hierarchy in mathematics. Students can develop good conceptual understanding of the derivative once they have a good conceptual understanding of the limit process and a good conceptual understanding of functions. However to compute the derivative of a particular function they have to develop some techniques. In the case of the derivative different techniques are needed because it is, in most instances, a far too complex a human task to use the limit definition. However this is certainly not the case for a software such as Maple (web reference 4). In fact in my classes, when computing the derivative of a function using Maple, I do not use the diff (differentiation) command but always use the limit definition. What new understanding do students develop from the differentiation techniques? They may help them to revise some of their concept knowledge of functions but they rarely help them to develop new conceptual understanding. The MICA Program has taken some tentative first steps at exploring the roles of these conceptual and technical hierarchies in undergraduate mathematics.

At Brock the technological environment in the mathematics courses involves a number of approaches. The lectures are held in smart classrooms with easy access to the appropriate technology. Instructors tend to use a mixed medium approach, involving board work, overheads, presentation softwares and the mathematical softwares used in the course. Some of the courses have scheduled laboratory sections while, in others, students access the laboratories in their own time or use their own computers. Assignments generally require both written and computer work. Maple is used extensively in the first year calculus and linear algebra courses and it is the development tool for many of the upper year courses. In the first year calculus course instructors and students use *Journey Through Calculus* by Bill Ralph(1999) – a calculus multimedia learning environment that develops mathematical concepts in a systematic way. At the start of any module the learner is given a verbal explanation of the contents of the module and is then engaged in an interactive activity or a visual presentation that focuses on the calculus concept to be developed. This is followed by a mathematical development of the concept, some applications and exercises. The final activity is one of assessment that challenges the learner to improve, to try questions newly generated each time. *Journey* uses Maple as its engine for solution and feedback to the learner.

The first two years of MICA courses presently use Visual Basic (web reference 5) as the programming language. Because I had the software on my laptop, I was able to project examples of student MICA course work during my ICME 10 presentation. Unfortunately there are only a few examples of student work on the MICA website (web reference 6). These examples date back to the first offering of the course when Java (web reference 7) was used as the programming language. The change to Visual Basic was made because students find it easier to learn and are therefore able to achieve more in their projects. In Appendix A I report on a project by two future teachers in the first year MICA course. No amount of written explanation can do justice to the dynamic and interactive environment they have created.

In some of their mathematics courses future teachers use softwares that are licensed by Ontario for use in all its schools. In a geometry course students use Geometer's SketchPad (web reference 8). Ryan Vandenberg, a future teacher, used it to developed an interactive set of geometry lessons for the middle grades (web reference 9). He has tailored each lesson to meet the requirements of the Ontario Curriculum guidelines. Some of his lessons are specially structured to allow the student to explore a particular geometric concept and to make conjectures, in other words these lessons are student directed. Other lessons can be used in a more teacher directed way. In another course that explores mathematics teaching and learning students use Fathom (web reference 10), a dynamic Data Management and Statistics program. The example in Appendix B provides a static view of the use of Fathom to develop the concept of regression. In this program students get a striking visual representation of the Least Squares method and are challenged to minimize the sum of squares for any set of points. It is a powerful environment for students to start thinking about the effect of outliers. The most recent Ontario secondary school mathematics curriculum (web reference 11), includes a final year applications and modelling course on the mathematics for data management. The course code is MDM4U. The course is built around a major student project, the first course of its type in any Ontario mathematics curriculum. For this reason it was recognized that the course would be a challenge for most mathematics teachers. Prior to the first offering of the course, the author with members of the Mathematics Education Forum of the Fields Institute (web reference 1) developed an internet site for teachers. How does one structure an internet resource that reflects modelling apprenticeship and the development of a major project? The following strategy was used: A future mathematics teacher in the concurrent education program was hired for a summer. Her responsibility was to work through the MDM4U course curriculum using the three Ministry approved texts. She was to develop a major modelling project of her choice that touched on most sections of the course. She was to record all her work. The outcome (web reference 12) demonstrates that modelling is a messy enterprise with false start, insufficient information and data, questions too general to be answered, etc. etc. The overriding message to the teachers is that students must start their modelling project early in the term and that they must continue to work at it throughout the course. The course MDM4U has now been offered for two years and teachers have sent in exemplars of work done by their students. These have been posted on the site.

In the summer of 2003 funded by EDUSOURCE (web reference 13) and other Brock University grants, a group of professors, future teachers and computer science students was assembled to develop a number of mathematics learning objects. Wiley (2000) provides the following description of learning objects "… the fundamental idea behind learning objects: instructional designers can build small (relative to the size of an entire course) instructional components that can be reused a number of times in different learning contexts. Additionally, learning objects are generally understood to be digital entities deliverable over the Internet, meaning that any number of people can access and use them simultaneously (as opposed to traditional instructional media, such as an overhead or video tape, which can only exist in one place at a time)." The aim of the group was to take a mathematical concept and to develop an engaging computer environment where students could learn and experience the concept. The group met with a number of elementary and secondary teachers to generate ideas of the kinds of mathematical concepts their students were having difficulty with, and also, to explore whether these teachers would use any of the learning objects in their classroom. A number of mathematical topics were selected and the students (all studying to become teachers), developed a Power Point presentation that outlined, in detail, each of the screens that were to be programmed. Some of the general guidelines for the learning objects were; each learning object should have an initial engaging activity (a game or a challenge); the user should never be in doubt what to do next; the number of words on each screen should be kept to a minimum; the number of screens should be small (less than 15); and the mathematics should be developed in a fun and interactive way. The faculty or student having completed a learning object development, made their Power Point presentation to the whole group, changes were suggested and a new presentation date set. Once a version was accepted by the group, it was given to a computer programmer. From now on changes were only allowed if the programmer was unable to program what was detailed in the Power Point information. In fact few changes were made because the computer programmers were part of the group approving the final version and they were sufficiently knowledgeable to suggest changes before the programming stage. It is natural to want to produce the best learning object possible. Unfortunately if changes are allowed at any stage an object can be in a constant state of flux and never be completed. Creators of the objects found this rule particularly difficult, nevertheless, it was a major factor that allowed the group to achieve much more than originally projected. The results of this project are six mathematics learning objects at different levels of education. I hope that you will try them out by accessing the Department of Mathematics site (web reference 14). Feedback to-date indicates that students get immersed in the mathematical activities.



**Conclusions**

This paper presents some innovative approaches to the use of technology in undergraduate mathematics education, and proposes ways to engage future mathematics teachers in reflecting on the teaching and learning of mathematics early in their education. As technology evolves so will its capacity to do more and more of the well structured components of mathematics. Therefore, the challenge for mathematics departments is to review their undergraduate programs in order to prepare their graduates for the 21st century. They may take their cue from Conway (1997). His article concerns the uncertain future of today's mathematics departments, and he writes:

> "We have to embrace technology, I don't mean just tolerate it; embrace it and celebrate it…. The professional mathematics community must adapt and learn how to best incorporate technology into instruction. Wits the existence of powerful, inexpensive computers, I see mathematics departments rethinking their entire curriculum…. Otherwise we are out of business…."

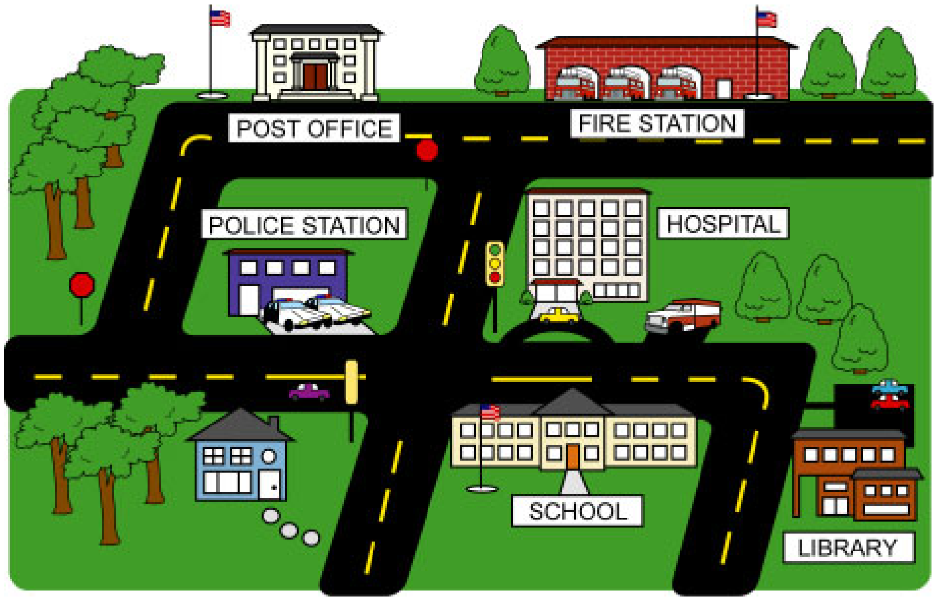
The challenge for mathematics educators is to review their programs for future teachers, in order to provide a breadth of experience in mathematics, to integrate the use of technology in a pedagogically sound way, and to reflect on its impact on the teaching and learning of mathematics.

**Appendix A**

***First year MICA project developed by Danielle Brignull and Katherine Christian***

In this appendix I provide a glimpse of an interactive environment developed by two students in their first MICA course. Their aim is to introduce exponent laws to those who have never seen them or worked with them before. No written report can do justice to this interactive environment. I shall mainly draw from the students' report as this highlights their maturity of approach to the teaching and learning of mathematics and the pedagogical use of technology.

A warm welcome to Mathville is extended to the user as she starts the program, sees the screen below, and is asked for her name.




The user is directed to the School to receive an introduction to the exponent laws. The authors are aware of different learning styles and note "each law is explained and described in words, outlined in general and shown in an example." They are also aware where this topic falls in the curriculum and note that "Under the new curriculum in Ontario, a teacher must grade under four categories, Knowledge and Understanding, Communication, Reasoning and Thinking and Application. This program contains elements from each category", and they go on to list where these can be found as the user accesses the various sites of the Post Office, the Police Station, etc.

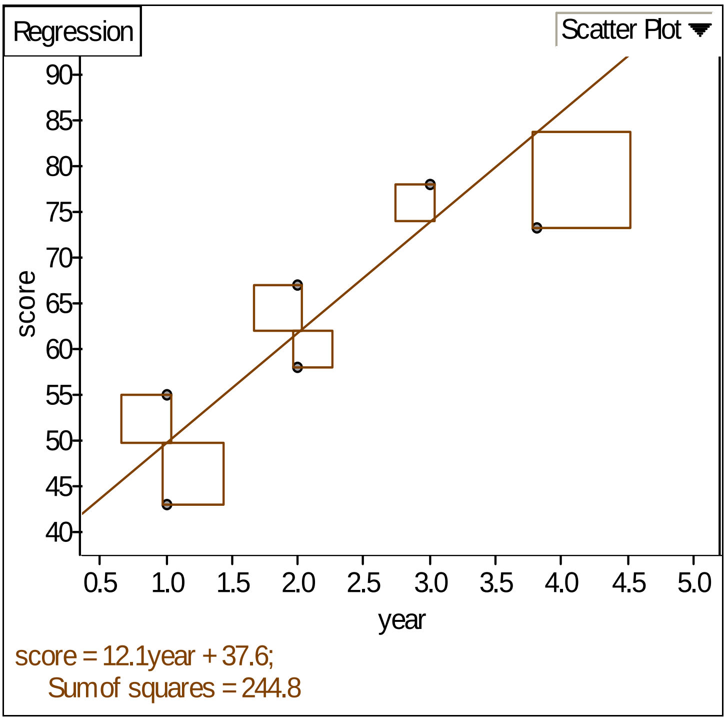
The authors point to the following highlights of the environment that they have created. The first is the ability to move between verbal, algebraic and numerical representations. The user is able to practice any of the laws "through an infinite number of randomly generated questions". Another feature is that the authors have, for some tasks, introduced a timer. Users also have access to a web based calculator. The authors use many different pictures throughout the activities, these introduce an unexpected element of fun.

Danielle and Katherine, have shown great maturity in their approach to pedagogical issues, and this is in a first MICA course. One can only imagine the depth of understanding that they will develop in the teaching and learning of mathematics and in the use of technology.

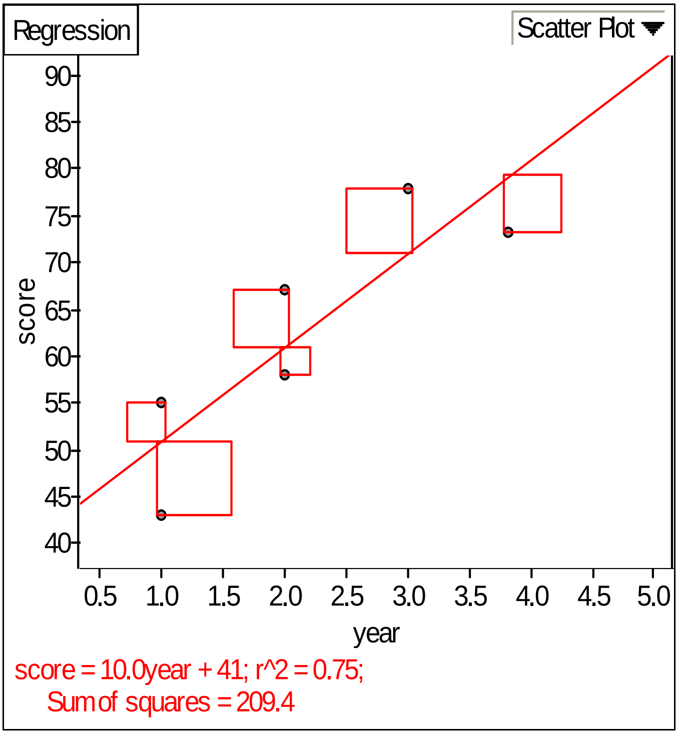
## Appendix B

***Static representation of a dynamic Fathom example developed by Brian Calvert***

In this example the user is asked to change the slope and intercept of the line so as to generate the minimum sum of squares. This is done simply by selecting the line in two different areas, moving one changes the slope, moving the other changes the intercept. With two parameters to play with this can be quite a challenge. A description follows the activity.




The points show how scores in a Math contest depend on year for 6 university students.
The vertical lines joining data points to the diagonal represent residuals.
Think of them as rubber bands that pull harder when they are stretched.
Then the squares represent potential energy.
Try to make the brown line fit the points as closely as possible.
Move the brown line up or down by clicking on its center. Rotate it by clicking near its end.
You are trying to make the sum of areas of the squared residuals as small as possible.
Compare to the least squares line in red, which minimizes potential energy and balances the up and down forces on both ends.




The user is then challenged to match the computer generated Least Squares line.

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Web reference 4: http://www.maplesoft.com/http://brocku.ca/ctl/pdf/Brock_Teaching_1.pdf site accessed October 15, 2004.

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Web reference 10: http://www.keypress.com/fathom/ site accessed October 15, 2004.

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