

# State-of-the-art in mathematical beliefs research

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## Abstract

*Teachers and pupils seem to have, in some cases, strikingly different views of mathematics, and of mathematics teaching and learning. The state-of-the-art within the constructivist view of learning has led researchers in mathematics education to investigate teachers' and pupils' mathematics-related beliefs and their implication for mathematics teaching and learning. Beliefs seem to be situated in the "twilight zone" between the cognitive and the affective domain, and thus, have a component in both domains. In order to sharpen the research being done, subcategories of teachers' and pupils' beliefs have been distinguished. For example, beliefs about mathematics education, beliefs about self, and beliefs about the social context.*

*Research done in different areas of beliefs is uneven in the sense that in some areas researchers have not yet been able to produce effective answers, whereas in other areas some clear influence of research on teaching practice can be seen. An open issue is, for example, changes in teachers' beliefs, whereas an example of a well-documented content area is gender differences. Additionally, we will discuss briefly the content of the recent Kluwer book on beliefs research (Leder et al. 2002) and furthermore show, by way of an example, the state-of-the-art in beliefs research in one country.*

*A preliminary comment:* In the following, I will give an overview on the state-of-the-art in beliefs research in the world. But I realise that such an attempt cannot lead to a complete view, since the presenter's basic beliefs and values form the framework within which he is able to work and form his overview. Therefore, the situation might seem totally different with the eyes of a researcher from another country or even more from another continent.

## Introduction

In the industrialized countries, everybody seems to know what mathematics is. But when the question is actually asked, one gets different answers depending on the respondent in question. School children understand mathematics differently from their mathematics teachers, and teachers of other subjects will explain it differently still. Yet another description is received, e.g., from a "man-on-the-street". And mathematics professors have their own view of mathematics.

The big variety of answers to the question "What is mathematics?" indicates that there is not only one understanding of mathematics, but several different views of mathematics (see later). And what is more important, there is not only one *right* view

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of mathematics while the others are *wrong*. Philosophers of mathematics (e.g. Ernest 1991, Hersh 1997) have introduced several *right* views of mathematics, and the idea is also accepted among mathematicians. This state of art within the constructivist view of learning has led researchers of mathematics education to investigate teachers' and pupils' views of mathematics and their implication for mathematics teaching and learning.

### What are beliefs?

The affective domain was for a long time a neglected area in research of mathematics education. About three decades ago an individual's attitude toward mathematics was brought up as one of the central research topics in mathematics education. The well-known Fenneman-Sherman attitude scale (Fennema and Sherman 1976) represents this phase. One may state that McLeod and Adams initiated a new phase in research of the affective domain in mathematics with their famous book (McLeod and Adams 1989), and the constructs have been further elaborated in recent research (e.g. Goldin 2002, Hannula 2004). The focus of research has changed from broadly defined attitudes to more specific sub-concepts: emotions, narrowly defined attitudes, values, and, most commonly, beliefs. Within beliefs research in mathematics education, further subcategories have been distinguished. For example, 1) beliefs about mathematics education (mathematics as a subject, mathematical learning and problem solving, mathematics teaching in general), 2) beliefs about self (self-efficacy, control, task-value, goal-orientation), and 3) beliefs about the social context (social and socio-mathematical norms in the class) (Op't Eynde et al. 2002). Another direction of sharpening has been within mathematics itself. Today we are no more considering attitudes or beliefs toward mathematics as an entity, but researchers distinguish, e.g., attitudes or beliefs on geometry or on problem solving.

Within school research the prevalent understanding of learning has emphasized cognitive academic achievements. Affective side-effects that are in connection with an individual's meta-cognitions, however, determine, to a large extent, how good a learner of mathematics one has become. Especially during the last decade, researchers around the world have paid more and more attention to mathematics learning as a process. Such a view highlights the importance of meta-cognition and affect, especially in the form of teachers' and pupils' beliefs. Beliefs seem to be situated in the "twilight zone" between the cognitive and the affective domain, and thus, have a component in both domains.

Individuals continuously receive perceptions from the world around them. According to their experiences and perceptions, they make conclusions about different phenomena and their nature. An individual's personal knowledge that includes one's beliefs is a compound of these conclusions. Since beliefs seem to be formed and changed in the social environment, persons compare these beliefs with their new experiences and with the beliefs of other individuals. Thus their beliefs are under continuous evaluation and change. When one adopts a new belief, this will automatically form a part of the larger structure of one's personal knowledge, of one's belief system, since beliefs never appear fully independently (cf. Green 1971). Thus, the individual's belief system is a compound of one's conscious or sub-conscious beliefs, hypotheses or expectations and their combinations.

Although beliefs are popular as a topic of study, the theoretical concept of "belief" has not yet been dealt with thoroughly. The main difficulty has been in the inability to distinguish beliefs from knowledge, and this old question does still not seem to be clarified (e.g. Abelson, 1979, Thompson, 1992). These problems are



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discussed e.g. in a recent paper of Pehkonen and Pietilä (2003). Another difficult issue has been the distinction between beliefs and other affective variables such as emotions, attitudes, and values. These variables have often been located in a continuum, stretching from intense and fluid emotions to stable and cool beliefs (McLeod, 1992). Such a view, however, has also been criticized, for example, for exclusion of mild or long-lasting emotional states and emotions that arise regularly (Evans et al. 2003).

As an implication of this fuzziness in the definition, one might mean different things with beliefs, depending on the discipline at issue and on the researchers who deal with them. For example, beliefs are considered equal to concepts, meanings, propositions, rules, preferences or mental images (Thompson, 1992). On other occasions, beliefs are seen in a much broader sense as "mental constructs that represent the codifications of people's experiences and understandings" and that shape their perception and cognition in any set of circumstances (Schoenfeld, 1998, 19). In social psychology the impressions of and reactions to other people are typically divided into beliefs, expectations and attitudes. There, beliefs are statements thought to be true, whether or not they are. Expectations are explicit or implicit predictions about people's future behaviors, and attitudes are emotional reactions to them. (Brophy and Evertson, 1981) These questions of fuzziness in defining beliefs are dealt with more broadly e.g. in the paper of Furinghetti and Pehkonen (2002).

On one hand, beliefs can be thought to form one part of an individual's meta-cognition (e.g. Schoenfeld, 1987). On the other hand, attempts have been made to define beliefs i.a. via attitudes (e.g. Törner and Grigutsch, 1994). In the different definitions, a truth-value is usually attributed to a belief (i.e. beliefs are a kind of knowledge), and the definitions are often seen to include an emotional component. Every now and then the definitions also say something about the stability and intensity of beliefs and about the nature of their origin or warranty. Here an individual's *beliefs* are understood in a rather wide sense as his/her subjective, experience-based, often implicit knowledge of and emotions about some matter or state of affairs. Such a characterization is very near the one given e.g. in the paper by Lester et al. (1989). Thus, the individual considers his/her beliefs to be true, not necessarily being so in reality (cf. Brophy and Evertson 1981). In the literature, the term *conception* is often used in parallel with beliefs. In the case of *conceptions*, the cognitive component of beliefs is stressed, whereas in basic (primitive) beliefs the affective component is emphasized. Therefore, an individual's conceptions are usually conscious, and he/she is able to reason about them.

The spectrum of an individual's beliefs is very wide, usually being grouped in clusters. Belief clusters continuously influence each other. Some beliefs depend on other, for the individual more important, beliefs. Here Green (1971) uses the term "the quasi-logical structure of beliefs" which means that the individual him- or herself defines the ordering rules. Thus, beliefs form belief systems that have a quasi-logical structure, and that might – or might not - be in connection with other belief systems. Therefore, the term *belief system* is used as a metaphor to represent how the individual's beliefs are structured. The affective dimension of beliefs influences the role and meaning of each belief in the individual's belief system.

### Results of beliefs research

In the literature, one can find several overviews of beliefs research (e.g. Underhill, 1988a; Underhill, 1988b; Schoenfeld, 1992; Thompson, 1992; Pehkonen, 1994; Pehkonen and Törner, 1996; Opt' Eynde et al. 2002). The continuing interest in beliefs

shows that there are still many unanswered questions. Recently the very first book on mathematical beliefs research (Leder et al. 2002) was published, in order to give an overview on different research perspectives and on research done so far.

There are some attempts to single out the role of research in the affective domain of mathematics. When considering research in his chapter on affect and mathematics, McLeod (1992, 575) noted: “Although affect is a central concern of students and teachers, research on affect in mathematics education continues to reside on the periphery of the field”. Additionally Schoenfeld (1992) states that there is much research done on students’ beliefs, but not so much on teachers’ beliefs. Furthermore, Pehkonen and Törner (1996) tried to sketch the range of beliefs research by plotting a sample of the published papers into a matrix form.

Within the last twenty years, mathematics-related beliefs research has been unevenly distributed across different continents and countries. Since its roots can be seen in the United States, the largest part of research has, understandably, been conducted there. But the idea of the importance to do research in this topic has spread all over the world, from the Anglo-American countries to many others.

In the following I will give some examples of the research accomplished in different countries, and how research papers has been distributed a cross the world. In the first place I try to illustrate by means of examples the wide range of mathematics-related beliefs research.

### ***Research done within PME***

The International Group for the Psychology of Mathematics Education (PME) was established in 1976, and since then it has had an annual meeting in different parts of the world. The conference is open to everybody, but is based on research reports (max 8 pages) that are blind reviewed beforehand, and the reports accepted are printed in the conference proceedings. Therefore, these conference proceedings that today are huge (more than 1000 pages, and with more than 100 research reports) give a good overview of important parts of the latest research done in mathematics education around the world.

My method is to look through the PME proceedings of the last ten years. Thus, I looked at the titles of the research reports, and focused on the following key words: the affective domain / attitudes, beliefs, conceptions, and teacher change. Therefore, my overview cannot be complete, but will, nevertheless, give a sufficient insight. In ten years of PME proceedings (1995–2004), I found 131 hits distributed as follows: the affective domain / attitudes (19), beliefs (47), conceptions (44), teacher change (21). In many countries, there were only a couple of hits, and therefore, I have listed in Table 1 the eleven largest frequencies, having four as the lowest acceptable frequency.

Australia	20
USA	17
UK	14
Cyprus	11
Finland	9
Israel	9
Brazil	6
Greece	5



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Italy	4
Mexico	4
South Africa	4



Table 1. Distribution of belief papers in the PME proceedings during 1995–2004.

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Of course, there are many other fora where researchers might publish their results, such as PME-NA (in North-America), MERGA (in Australia), etc. There are also other possible biases, as a researcher might actually investigate beliefs but uses some other term for it. In any case this list gives a rough overview of what is happening in beliefs research world-wide.

The very first observation is that beliefs research is done all over the world, on almost each continent – with Asia is an exception. It is interesting that the US is no more the quantitatively leading country. In Table 1, it is in the second place, but if we relate these figures with population size, the order is very different. Then Cyprus will be first one, Finland second, and Australia third.

In the following, I have arranged the countries according to continents, in order to give the reader a better overview. Only such researchers are mentioned who had more than one research report in the proceedings. Furthermore, beside the researchers from the PME proceedings I have mentioned some prominent researchers I know from other publications. And within each country the names of its researchers are usually given in an alphabetic order. But it is important to notice that in no way do I try to give a complete list.

*America.* In America, the most important country in this context is certainly the *United States*. The researchers found in these PME proceedings are Valerie DeBellis, Gerald Goldin and Paola Sztajn. Other well-known names in beliefs research are e.g. Deborah Ball, Tom Cooney, Lynn Hart, Peter Kloosterman, Douglas McLeod, Norma Presmeg, Alan Schoenfeld, and Erna Yackel. In *Canada*, the internationally most famous researcher might be Olive Chapman. Also in *Mexico* and in *Brazil* people are beginning to work actively on beliefs research; one internationally well known beliefs researcher from Brazil is Vania Santos.

*Europe.* Also in Europe beliefs research is spread unevenly. Therefore, I will give the countries in the alphabetical order, and mention only one or two researchers from each. *Cyprus* is the top country in beliefs research, and the pair of researchers Constantinos Christou and George Philippou seem to be the *primus motors*. In *Finland* Markku Hannula and Erkki Pehkonen have worked for a long time on beliefs. In *Germany* Andrea Peters and Günter Törner should be mentioned. In *Greece* the most well known beliefs researcher within the PME community is Kronos Kynigos. *Israel* has many active beliefs researchers, such as Dina Tirosh, Pessia Tsamir and Shlomo Vinner. In *Italy* Pietro di Martino and Rosetta Zan have done much research on attitudes. The representative in *Spain* might be Salvador Llinares. In the *UK* at least the names Paul Ernest and Celia Hoyles should be mentioned.

*Australia.* In *Australia* there are many internationally well-known beliefs researchers: Helen Forgasz, Gilah Leder, Beth Southwell, and Kaye Stacey. From *New Zealand* the name Kathy Irwin should be mentioned.

*Asia.* In Asia beliefs research has not yet spread so much as on the other continents. In China one name should be singled out in beliefs research, Thomas Wong. In Japan some of researchers are doing serious research in mathematics-related beliefs, but no name has yet gained international recognition.

*Africa*. In Africa, there are, especially in *South-Africa*, also some researchers in the field, but none of them is internationally well-known.

### **A book on beliefs research**

Another international view can be gained with the aid of the recently published book on mathematical beliefs research (Leder et al. 2002). This first book on beliefs offers an overview of different research perspectives and on research done so far. In the following, with the help of the chapters in the book, a kind of overview on the state-of-the-art in beliefs research in the world is given. It is understandable that such an attempt cannot lead to a complete survey, since the authors of the book form a small subset of the beliefs researchers in the world.

The book is based on the presentations at a specialist conference in Oberwolfach (Germany) in November 1999 about mathematical beliefs research (cf. Pehkonen and Törner 1999). The book consists of three parts: On the concept of belief (five chapters), on teachers' beliefs (six chapters), on students' beliefs (five chapters). Furthermore, each part has a summarizing chapter.

*On the concept of belief.* The first chapter begins with an overview of students' mathematics-related beliefs by *Peter Op't Eynde*, *Erik de Corte* and *Lieven Verschaffel* (Belgium). In two following chapters, characterizations of beliefs are considered. *Fulvia Furinghetti* (Italy) and *Erkki Pehkonen* (Finland) discuss possible characterisations of beliefs, based on results of a specialist questionnaire. *Günter Törner* (Germany) suggests a kind of an exact belief definition that is common in mathematics. *Gerald Goldin* (USA) combines in his chapter affect and beliefs. *Gilah Leder* and *Helen Forgasz* (Australia) deal with research methods on beliefs, additionally presenting a new method for information gathering.

*On teachers' beliefs.* Teacher change is a central topic here. *Melvin Wilson* and *Thomas Cooney* (USA) discuss beliefs generally from the viewpoint of teacher change. *Lynn Hart* (USA) presents results of a four-year follow-up study of teachers' beliefs. *Olive Chapman* (Canada) has observed a natural change in teachers' beliefs in high school mathematics. An attempt to change student teachers' beliefs is described by *Salvador Llinares* (Spain). *George Philippou* and *Constantinos Christou* (Cyprus) deal with teachers' efficacy beliefs on mathematics teaching. Teacher change is sought to be promoted through different interference actions, as *Gwendolyn Lloyd* (USA) stresses the role of the curriculum material.

*On students' beliefs.* *Peter Kloosterman* (USA) discusses students' beliefs about mathematics in secondary school. *Brian Greer* (USA), *Lieven Verschaffel* and *Erik de Corte* (Belgium) describe students' typical responses to word problems, and some factors behind such responses. *Norma Presmeg* (USA) singles out the gap between everyday and school mathematics, and offers some methods to bridge them. *Erna Yackel* and *Chris Rasmussen* (USA) present connections between beliefs and norms in the classroom. *Pessia Tsamir* and *Dina Tirosh* (Israel) push forward students' intuitive beliefs.

### **Some special cases of beliefs research**

Research done in different areas of beliefs is uneven in the sense that in some areas researchers have not yet been able to produce effective answers, whereas in other areas there is some clear influence of research to be seen. An open issue is, for example, change in teachers' beliefs, whereas an example of a well-documented content area is gender differences (cf. Leder et al. 2002). Furthermore, a variety of answers to the question "What is mathematics?" is discussed.



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### ***Teacher change***

A common feature for all published research results seems to be the observation that change is a very slow process, and that we don't have any method to accelerate it. Through research we understand the change mechanism better than earlier, but the main question (How can one accelerate the change?) is still unsolved in the sense that research has not found any recipe for change.

When discussing teacher change and the possibilities to influence it, the question arises, among others, what is cause and what is effect: For example, Ernest (1989) states that beliefs regulate a teacher's teaching practice in classroom. Furthermore, there exists evidence that changes in classroom practices might change also the teacher's beliefs (e.g. Guskey, 1986). This might be a similar circle as is formed by beliefs and learning in the case of pupils – beliefs influence learning, and learning affects beliefs – since teacher change can be considered as kind of learning (Nisbet and Ross, 1980). Another problematic observation is that in several implemented intervention studies with voluntary teachers only some teachers have changed their practice while many have not (e.g. Senger 1999; Borko et al. 2000; Wood, 2001; Hart, 2002). The influence of the intervention seems to stay only as long as the researcher is in the class (e.g. Cobb and al. 1990). When researchers leave the classroom, teachers fold back into their old practices, and teacher beliefs seem to be unaffected. Thus the question “How can we help all voluntary teachers to change themselves?” is also open.

A similar observation has been done and documented already about twenty years ago: In Cooney's study (1985), a teacher student (Fred) was interviewed about his teaching beliefs during his last study years, and was observed during his first school year. One could clearly see that he folded back into his old beliefs and habits during his first independent school experiences.

A skeptical question on change has been put forward i.a. by Leila Pehkonen (1991, 1994): Is change at all possible? Should we rather change the research question and put it other way round: What are the stable elements in school that teachers don't want to see changed? One of her research questions was as follows: “Which are the good and stable elements in teaching of school mathematics, from the view point of teachers?” Based on thematic interviews, she concluded that the desire to be a respectable teacher lies behind teachers' answers.

### ***Gender differences***

Research on mathematics and gender has been singled out as one area where “some aspects of beliefs about self have been researched quite thoroughly” (McLeod, 1992, 580). Although gender differences in mathematics achievement have been declining and even disappearing in some cases (e.g. Fennema and Hart, 1994, Beaton et al., 1997), there are still clear gender differences in beliefs, attitudes and choosing of non-compulsory courses in mathematics (e.g. Fennema and Hart, 1994; Leder et al., 1996).

The strongest gender difference in mathematical beliefs has been the fact that boys perceive mathematics as a male domain, whereas girls see mathematics as more or less gender neutral (Frost et al., 1994). Another clear gender difference is that boys have higher self-confidence in mathematics than do girls do (e.g. Bohlin, 1994; Leder, 1995; Hannula and Malmivuori, 1997; Maijala, 2002). Girls are also more prone to mathematics anxiety than boys (Hembree, 1990; Frost et al., 1994).

This knowledge on gender differences in beliefs has led to some changes in national policies of education. For example, in Australia (Blackmore et al., 1996) and in Great Britain (Harding and Parker, 1995) project work was introduced as an

element in national examinations at least partially to increase gender neutrality of the testing system.

### ***What is mathematics?***

As mentioned in the beginning, researchers have developed, in the philosophy of mathematics education, many alternative answers to the question “What is mathematics?” in the sense that there are several *right* answers, and none of the answers can be considered as totally *wrong* (e.g. Ernest, 1991; Hersh, 1997). Here I will introduce results of two such studies: Three aspects of mathematics, by Törner and Grigutsch (1994), and fourteen different types of understanding of mathematics by Mura (1995).

Combining the categories of Dionné (1984) with the groups used by Ernest (1991), Törner and Grigutsch (1994) developed and used the following three categories for aspects of mathematics:

A *“Tool box”*-aspect = mathematics is a collection calculation rules and procedures which will be applied according to the situation.

B *Systems*-aspect = mathematics is a formal system within which one acts rigorously, logically and exactly.

C *Process*-aspect = mathematics is a dynamic process in which everybody creates his own mathematics according to his needs and abilities.

People who are involved in applying mathematics are mainly proponents of the first aspect (tool box). The second aspect (system) is typical of those teaching at departments of mathematics. The third one (process) is the way in which professional (active) mathematicians consider their research domain. Usually people have a mixed view of mathematics, in the sense that their view is a compound of all these three aspects. Only the weight of each aspect varies from person to person. Methods to find out a person’s share on each aspect are discussed in the study by Pehkonen and Törner (2004).

There are only a few studies published on the conceptions of mathematics professors. For example Roberta Mura investigated professional mathematicians’ (mathematics professors, N=103, and mathematics educators, N=51) views of mathematics in Canada (Mura, 1993, 1995), using the open question “What is mathematics?”. She could group the answers into fourteen different types of understanding of mathematics (Mura, 1995, 389-391):

1. The creation and study of formal axiomatic systems, of abstract structures and objects, of their properties and relationships
2. Logic, rigour, accuracy, reasoning, especially deductive reasoning; the application of laws and rules
3. A language, a set of notations and symbols
4. Design and analysis of models abstracted from reality; their application. A means of understanding phenomena and making predictions
5. Reduction of complexity to simplicity
6. Problem-solving
7. The study of patterns
8. Inductive thinking, exploration, observation, generalization
9. An art, a creative activity, a product of the imagination; harmony and beauty



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10. A science: the mother, the queen, the core, a tool of the other sciences
11. Truth
12. Culturally determined content (ethno-mathematics)
13. Reference to specific mathematical topics (number, quantity, shape, space, algebra, etc.)
14. Other (difficulty, impossibility, futility of defining mathematics, circular definitions, etc.)

Mura's (1995) study shows how huge variation that there is also in mathematicians' understanding of mathematics. Their answers depend, probably, heavily on their interests in mathematics, on their mathematical specialization, on the problems they are just pondering on for the time being.

### **An example of beliefs research - Finland**

In the beginning of the 1990's, research on mathematics-related beliefs and conceptions started in Finland (cf. Pehkonen, 1993). There were several enterprises, all financially supported by the Academy of Finland that promoted the development: In the middle of the 1990's, the Finnish-German co-operational research enterprise MAVI (= Mathematical Views) was established. A national graduate school for mathematics, physics, and chemistry educators was initiated. Several research projects on mathematical beliefs (by the author) were accepted and financed.

Just recently I wrote, together with Markku Hannula, a paper on beliefs research in Finland (Pehkonen and Hannula, 2004) where one may find an enlarged version of this article.

### ***MAVI Activities***

Beliefs research in Finland was strengthened, when the Finnish-German co-operational research enterprise MAVI (= Mathematical Views) was launched in the middle of the 1990's. MAVI activities were financed by the Academy of Finland and DAAD (= Deutsche Akademischer Austauschdienst) for three years 1996–98. The main activities were joint workshops in both countries, where especially young researchers were supported. From these workshops seven reports were published in the publication series of the University of Duisburg and the University of Helsinki. Among the Finnish MAVI participants, there were mainly doctoral students, but also some experienced researchers, such as Sinikka Lindgren (Lindgren, 1995) from the University of Tampere participated.

Gradually the MAVI workshops became so popular that some foreign researchers regularly took part in them. And at the end of the 1990's, the MAVI workshops grew into a joint European enterprise, which can be seen e.g. in the latest published MAVI-11 report (cf. Di Martino 2002). The MAVI meetings offer a safe environment for doctoral students to present their ideas and to get feedback in English.

Another consequence of the Finnish-German joint MAVI activities was an international specialist conference in Oberwolfach in November 1999 that was also initiated and organized by Pehkonen and Törner (cf. Pehkonen and Törner 1999). In the Oberwolfach Institute, a group of about twentyfive selected international specialists in beliefs research discussed for several days their understanding of beliefs and related topics. This meeting resulted the recently published book on beliefs (cf. Leder et al., 2002) outlined above.

### ***Finnish Dissertations on Mathematical Beliefs***

Dissertations in Finland that were produced more or less with the help of MAVI activities are by Kupari (1999), Huhtala (2000), Malmivuori (2001), Pietilä (2002a), Soro (2002b) and Hannula (2004). But there are also Finnish dissertations on mathematical beliefs that have been accomplished totally outside of the MAVI environment: Kaasila (2000) and Perkkilä (2002).

When we take a closer look at the topics of the dissertations, teachers' beliefs seem to be the most popular domain: Pekka Kupari (1999) charted teachers' beliefs through a survey study trying to sketch a picture of their views of mathematics. Raimo Kaasila (2000) and Anu Pietilä (2002a) used qualitative methods to follow the development of students' beliefs in elementary teacher education (see also Kaasila, 2002 and Pietilä, 2002b). Päivi Perkkilä (2002) combined quantitative and qualitative methods to sketch firstly a general outlook of in-service teachers' views on mathematics in elementary school, and then deepened the approach through interviewing a group of teachers. Riitta Soro (2002b) developed a new instrument to quantitatively measure mathematics teachers' gender-related beliefs, and administered a survey on mathematics teachers in lower secondary school (see also Soro, 2002a).

Sinikka Huhtala (2000) used a grounded theory approach to explore what mathematics means to those nursing students who have difficulties in meeting the formal requirements of their schooling in drug calculations (see also Huhtala, 2002). Marja-Liisa Malmivuori (2001) accomplished an extensive exploration of the literature, and made a theoretical synthesis of the dynamics of affect, cognition, and social environment in self-regulation of learning thus providing a larger context of theory to embed beliefs research into. Markku Hannula (2004) developed certain aspects of a theory on the origin and change of pupils' mathematical attitudes and beliefs.

Summarizing, the Finnish research has contributed especially to understanding the development of beliefs during teacher education and to the development of theoretical foundation of beliefs research.

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