

Klein and Freudenthal

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Introduction

It was a great honour to be invited by the Programme Committee to speak about Klein and Freudenthal, two men whose achievements have been commemorated through ICMI's naming of medals in their honour. Yet it is not an easy task. Should I try to explain why, say, Klein, who served six years as President of ICMI, was more deserving of this honour than the Swiss mathematician, Henri Fehr, who was ICMI's General Secretary from its inception in 1908 until the Second World War sent it into hibernation, and who was from 1952 to 1954 its Honorary President? No, I do not want to get involved in ranking contributions. However, what is without doubt is that both Klein and Freudenthal were men who contributed considerably to mathematics, the study of the history of mathematics, and to mathematics education. Few, if any, would challenge the claim that no one involved with ICMI has attained such distinction as a mathematician and has contributed more to the growth of mathematics as did Klein¹. And no President has contributed more to the advancement of mathematics education and displayed the same degree of professionalism and involvement in this task than did Freudenthal. Should I, then, attempt to justify these statements in my talk? This would be impossible in 45 minutes. I intend, therefore, to give a short account of the careers of the two before looking specifically at their contributions to ICMI; how they found it, how they left it, in what ways ICMI has changed since their times, and, in particular, how ICMI might build on their hopes and objectives.

First, then, a brief account of Klein's career that will allow us to see his contribution to ICMI in context.

Outline Biography: Felix Klein

1849 25 April, born in Düsseldorf

1865 Obtained *Abitur* from his *Gymnasium* and entered Bonn University

¹ Any history of mathematics will display the great contribution that Klein made to the development of the subject during the years in which he was active in research. However, his influence on the growth of mathematics in the late 19th and early 20th century was immense. That Göttingen came to be seen as "the powerhouse" for mathematics was largely due to him and to his ability to attract to it and foster the work of mathematicians such as Hilbert. He was also largely responsible for the way in which *Mathematische Annalen* rose to challenge Crelle's *Journal* as the key research periodical of its day. Schubring (1989) gives references to works that study his influence on the development of German universities. Some idea of his standing in other countries is given by his being elected a Fellow of the Royal Society (of London) in 1885, being awarded the De Morgan Medal of the London Mathematical Society (the highest award within British Pure Mathematics) in 1893, and the Copley Medal (the Royal Society's premier award) in 1912. His ability still to inspire mathematicians was demonstrated by the publication in 2002 of *Indra's Pearls: the vision of Felix Klein* (D. Mumford, C. Series, D. Wright; Cambridge University Press) in which the authors used computers to develop work on symmetry that Klein had begun laboriously by hand.



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- 1866 Physics laboratory assistant to J. Plücker²
- 1867 *Dr. phil.* in Bonn
- 1868 Studied in Göttingen with Clebsch³
- 1869 Visited Paris (and again in 1870 with Sophus Lie⁴)
- 1871 *Habilitation*. Lecturer in Göttingen.
- 1872 Appointed Professor at Erlangen University (*Erlanger Programm* and the *Erlanger Antrittsrede*⁵)
- 1875 Appointed Professor at the *Technische Hochschule*, Munich
- 1880 Appointed Professor of Geometry at Leipzig University
- 1881 Severe illness which cut short his career within mathematical research
- 1886 Appointed Professor at Göttingen
- 1895 Doctorate awarded to his research student, Grace Chisholm (Young): the first award of this nature in Prussia to a woman⁶
- 1900 *Gutachten* (an outline programme for the reconstruction of secondary and higher education)
- 1905 *Merano Lehrplanentwürfe*
- 1908 Appointed First President of ICMI
- 1908 Published *Elementarmathematik vom höheren Standpunkt aus* – 09 (*Elementary Mathematics from an Advanced Standpoint*)
- 1913 Retired from Göttingen
- 1925 22 June, died in Göttingen.

² Plücker (1801-68) was an extremely influential physicist who originated the idea of spectrum analysis and in 1859 discovered cathode rays, but whose interests later turned to geometry. Like Klein, he was elected a member of the Royal Society (1855) and received its Copley Medal (1866).

³ Clebsch (1833-1872) was a mathematician (one of the founders of modern algebraic geometry) and physicist with whom Klein studied after Plücker's death, and who supervised Klein's *Habilitation* (the qualification then required to give lectures). Clebsch was one of the founders of the journal *Mathematische Annalen* of which Klein became editor-in-chief in 1876.

⁴ Lie (1842-1899) and Klein were greatly affected by their visit to Paris and their meetings with Camille Jordan (1838-1922) who, together with Lie, introduced Klein to many ideas in group theory. There is here a link of interests with Freudenthal who once told me of how a similar visit to Paris (but then to the school of Jordan) introduced him to algebraic ideas and methods of approach that he had not met in Berlin. Later, in the 1950s, Freudenthal was to publish important papers on Lie groups and went on to write two books on them: *Lie Groups* (New Haven, Yale University Press, 1961) and, with H. de Vries, *Linear Lie Groups* (New York, London, Academic Press, 1969).

⁵ Klein's "Erlanger Programm" which framed geometry in group-theoretic terms is well known. However, in his actual inaugural lecture (*Antrittsrede*) Klein discussed his guiding principles for the reform of mathematics education (see Rowe, 1985).

⁶ Grace Chisholm (whose thesis was on mathematics) described Klein's attitude to women students as follows: "He will not countenance the admission of any woman who has not already done good work, and can bring proof of the same ... and [until] he has assured himself by personal interview of the solidarity of her claims. [His] view is moderate. There are members of the faculty who are more eagerly in favour of the admission of women and others who disapprove altogether". (Quoted in Cartwright, 1944).

In 1911, Klein went on to supervise the first *Habilitation* on mathematics education (see Schubring, 1988). It is significant that his student, Schimmack, should have qualified as a university lecturer, not by following up his doctorate in mathematics with more evidence of his ability as a mathematical researcher, but rather by demonstrating his appreciation of the problems of teaching mathematics. The first Ph.D.s in mathematics education were probably those awarded in 1906 in the USA to two students of D.E. Smith, who both wrote on historical topics (see Donoghue, 2001).



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What is immediately noticeable on looking at this brief outline of Klein's career is his great precocity: he achieved so much so quickly. What it only hints at is the broad conception of education that his training gave him. At Bonn he obtained an education, not only in mathematics, but also in physics, chemistry, botany, zoology, and geology (see Schubring, 1989). Moreover, within the mathematics course considerable emphasis was placed on the teaching of applied mathematics (mechanics and statics)⁷. Appointments at Erlangen and Munich introduced him, not only to a different educational system – that of Bavaria – but also to two very different types of institution. Erlangen, a small, university offered him few opportunities to build a department: his students were few, never more than eight and at first only two (Schubring, 1989; Wussing and Arnold, 1989). In his inaugural lecture at Erlangen when only 23, however, he demonstrated that he already had strong views on education. He deplored the “fateful division” of education, in particular through the existence of different types of school at secondary school level. In later years, he was also to argue against the distinction drawn in higher education between universities and *Technische Hochschulen*. Views, incidentally, which he was to change. What were never to change, however, were Klein's beliefs in the need to stress the teaching of how mathematics is applied and to emphasise the unique role of mathematics as the fundamental scientific basis for technology: a role that was increasingly challenged in Germany in the 1870s and 80s (see Schubring, 1989). Klein's move to Munich was to provide him with greater opportunities and new challenges. There he was able to teach advanced mathematics courses to more appreciative and gifted students⁸, and his experiences in Munich were later to have an effect on ICMI's initial programme.

Leipzig University, at which Klein was appointed as a geometer, offered other new possibilities and saw his fame as a mathematical researcher further enhanced before, in 1882, when only 33, Klein became seriously ill and depressed. His “research career”, in the sense of published papers, virtually ended. In his own words, “I limited myself to ideas and guidelines and left the exact implementation and further development to younger forces who helped me stand aside”⁹. This did not prevent his appointment to Göttingen or to the widening of his influence throughout the whole mathematical world. In Göttingen, in addition to fostering the research of others, he lectured on a wide range of topics, mainly on the interface of mathematics and physics, e.g. mechanics and potential theory (see MacTutor), and published several influential books on both pure and applied mathematics¹⁰.

⁷ Klein's D. Phil supervisor, Rudolf Lipschitz, gave the mechanics seminars.

⁸ Among his many students – for now he often taught classes of 100 or more – were Planck, Runge and Hurwitz (whom Klein supervised for his doctorate) (see MacTutor).

⁹ “Ich beschränkte mich auf Ideen und Rechtenlinien und überliess die genaue Durchführung und weiter Ausgestaltung den jüngeren Kräften, die mir helfend zur Seite standen.” (Klein, 1923, p. 10).

¹⁰ For example (quoting an English translation where known, and the original German dates of publication):

On the development of mathematics in the nineteenth century (Berlin, 1926-7), Brookline, Maths Sci. Press, 1979.

Famous Problems of Elementary Geometry, Boston, London, Ginn, 1887.

On the ikosahedron and equations of the fifth degree (Leipzig, 1884), London, Trübner & Co., 1888.

The mathematical theory of the top, New York, Scribner's Sons, 1897.

Vorlesungen über die Theorie des Kreisels [gyroscopes] (with A. Sommerfeld), Leipzig, 4vols, 1897-1910.



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But what can be said of Klein as a mathematics educator? There is no obvious biographical source that deals specifically with this and I am greatly indebted to Gert Schubring for spelling out to me what he considers to be Klein's major contributions. First, it must be stressed that in Klein's time mathematics education as a discipline was virtually non-existent. Klein, then, was, in much that he did, a pioneer, but one whose instinct was to act where he saw a need, not to engage in empirical research.

(1) In an educational system that contained a range of different institutions at the three levels – primary, secondary, and higher, he developed a clear understanding of their interconnectedness. Thus, for example, in 1900 the Prussian Ministry of Education specifically sought his advice on the secondary school preparation he thought necessary for admission to technical colleges. His response (*Gutachten* – expert witness) set out a possible programme for the reconstruction of secondary and higher education¹¹.

(2) He recognised the gulfs that existed between higher education and the schools, i.e. between “mathematics as a science” and “school mathematics”, and also that between primary and secondary school mathematics. It was to help bridge that first gulf that he wrote his *Elementary Mathematics from an Advanced Standpoint*. He had great influence on the national conference held in 1905 at Merano to discuss desirable developments in secondary school curricula. In particular he pressed the case for the teaching of the differential and integral calculus in schools.¹²

(3) He recognised the need for in-service training for teachers and in 1892 initiated summer schools for teachers. These were held every two years and Klein himself was a lecturer at them¹³.

(4) He was not concerned solely with students bound for higher education, but was active in improving mathematics teaching in the *Volksschulen* (elementary schools) and in the mathematical training of their teachers.

Outline biography: Hans Freudenthal

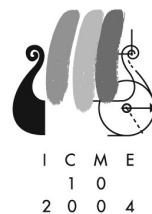
- 1905 17 September, born at Luckenwalde, Germany
- 1923 Entered Berlin University
- 1927 Met Brouwer and visited Paris
- 1929 Assistant in the Mathematics Department, Berlin
- 1929 Appointed Assistant-Professor to Brouwer at the University of Amsterdam
- 1930 Ph.D. at Berlin University
- 1936 Mathematics Study Group of the Dutch department of the New Education Fellowship (WVO)
- 1940 German invasion of the Netherlands: Freudenthal, a Jew, dismissed from his post

Encyclopädie der mathematischen Wissenschaften, Chief Editor and contributor to the volumes on Mechanics, Leipzig, Teubner, 1901-35.

¹¹ The response is reprinted in full (in German) in Schubring, 1989.

¹² The outcome of the Merano Conference was to make the teaching of calculus optional, but by 1910 most Prussian *Gymnasiums* taught it and teaching it became compulsory in 1925.

¹³ This was by no means the first vacation course for teachers: as long ago as 1766, Charles Hutton (Newcastle, England) advertised a Christmas vacation course for teachers as a private, commercial enterprise (see Howson, 1982, p.63). However, it was possibly the first within a state educational system.



- 1940-1945 In hiding and in prison (1944): began to write a “didactics of arithmetic”
- 1945 Liberation and resumption of duties at the University of Amsterdam
- 1946 Appointed Professor at the University of Utrecht
- 1957 Dieke and Pierre van Hiele awarded Ph.D.s
- 1967 Appointed President of ICMI
- 1968 *Educational Studies in Mathematics* first appears
- 1969 First International Congress on Mathematics Education (Lyon)
- 1970 Establishment of IOWO (The Institute for the Development of Mathematics Instruction)
- 1973 *Mathematics as an Educational Task* published
- 1974 Retirement from Chair at Utrecht University
- 1978 *Weeding and Sowing* published
- 1981 IOWO becomes part of the Faculty of Mathematics and Computer Science, Utrecht
- 1983 *Didactical phenomenology of mathematical structures* published
- 1990 *Revisiting Mathematics Education: China Lectures* published 13 October, died at Utrecht
- 1991 Freudenthal Institute named.

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Let us now leave Klein’s career for the moment and briefly consider that of Freudenthal. As we see, Freudenthal’s career proceeded at a more leisurely pace than did Klein’s. He studied at Berlin University for six years before being offered a position as assistant to Brouwer in Amsterdam¹⁴. This proved at least a temporary blessing, for had Freudenthal taken an academic post in Germany then he would, as a Jew¹⁵, have been dismissed from it once Hitler came to power. As it was, he was to lose his post in Amsterdam when the Netherlands was invaded in 1940. Like Klein, he faced a personal crisis in his 30s, but of a very different nature. In fact, it allowed Freudenthal to demonstrate his non-mathematical talents¹⁶ and also to develop his

¹⁴ That Freudenthal acted as an assistant in Berlin came to my knowledge in what was to me an interesting manner. Once, on tidying my university office, I came across some exercises that I had done at university and which had been corrected by my tutor, Walter Ledermann. On telling Professor Ledermann of this, he said that he still had some exercises from his days at Berlin University which had been corrected by an assistant named Freudenthal: did I know of him? An interesting account of Freudenthal’s student years in Berlin was published posthumously in 1998.

¹⁵ Klein’s racial ancestry was to become a matter of dispute in the Hitler years. In the fifty years following their emancipation, finally achieved in Prussia in the late 1860s (cf. the changes that allowed J.J. Sylvester to become an Oxford Professor in 1883), an increasing number of important posts in the German university mathematics and physics departments came to be held by Jews. This was abhorrent to the Nazi anti-Semites and a scapegoat, preferably Jewish, had to be found. Thus it was claimed that Klein, who had held such a dominant position in German scientific life, had a Jewish background; a claim repeated in the Nazi *Völkischer Beobachter*. In an attempt to rehabilitate Klein, in a manner in keeping with the times, the *Göttinger Tageblatt* assured its readers that Klein was a true Aryan and that rumours of his Jewish ancestry were due to the Jews falsely claiming him in order to increase their own prestige. The Nazi mathematician, Bieberbach, then supported Klein’s Aryan status and, furthermore, argued that his writings showed him to be a proto-Nazi thinker (see Rowe, 1986).

¹⁶ It is on these talents we shall now concentrate. However, his wide-ranging mathematical knowledge and interests must be stressed. For example, in addition to the books on Lie groups mentioned in an earlier footnote, Freudenthal also published books on logic (*The Language of Logic*, Amsterdam, Elsevier, 1966; *Lincos: design of a language for cosmic intercourse*, Amsterdam, North Holland, 1960) and one entitled *Probability and Statistics* (Amsterdam,



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RL

Regular Lecture

burgeoning interests in mathematics education. Freudenthal had by then gained a good command of Dutch and, indeed, he was a very gifted linguist¹⁷. How he began to write novels and in 1944 entered one for a competition under the name of a non-Jewish friend. It won, and the friend was then placed in the embarrassing position of having to explain how he came to write the book (see Bos, 1992)¹⁸. Even before the war, Freudenthal had begun to take an active interest in the problems of teaching mathematics in secondary schools and participated in the meetings of the mathematics group of the New Education Fellowship (WVO). During the war, when helping his two young sons to learn arithmetic, he was inspired to read anything that had been published on this topic. This, in turn, led him to begin to write a “didactics of arithmetic”. His 103 typed folio sheets still exist, but the work was never finished. Goffree (1992) tells us that Freudenthal’s attitude, as displayed in this draft, is distinguished by:

- his lack of regard for advanced statistical calculations based upon simple data;
- a mistrust of “quoted” research as opposed to looking at original sources;
- the need to demonstrate the applicability of a pedagogical method in practice;
- the need to distinguish between “number concept” and “concept of number”¹⁹;
- the benefits of teaching in small groups of 3 to 6 pupils.

Such beliefs were *not* to change over the years!

Another key step in Freudenthal’s educational development came with his supervision in the 1950s of the two van Hieles, Dieke, the practitioner, and Pierre, the theoretician. Their “levels” of geometrical thinking are now established as part of mathematics education history and have acted as a great stimulus to further research and theorising. Clearly the van Hieles would have gained much from their supervision by Freudenthal, but he, for his part, always acknowledged what he himself had learned through this process. What is remarkable, though, is that, in the 1950s, a professor of mathematics, still at the peak of his research career, should have spent time supervising research work in mathematics education²⁰.

By then Freudenthal was participating in educational meetings outside the Netherlands. However, as he frequently told me, he was to make one of the worst decisions he ever made in his life by not attending the OEEC (Organisation for

Elsevier, 1965). He also wrote a very successful “popular” book, *Mathematics Observed* (London, World University Library, 1967), and, over the years, many papers on the history of mathematics including a considerable number of essays on the work of leading mathematicians for *The Dictionary of Scientific Biography* (see MacTutor for a list of these). A semi-autobiographical book, *Schrijf dat op, Hans, Knipsels uit een leven* (Make a note of that Hans: cuttings from a life) was published in 1987 (Meuenhoff Informatief Amsterdam).

¹⁷ He once told me how, at one meeting, Dieudonné, of whose educational theories Freudenthal was a great opponent, once lost his temper with him and began shouting at Freudenthal. “Don’t shout at me”, Freudenthal had replied, “for I can shout louder than you – and in more languages!”

¹⁸ Freudenthal also was to write a newspaper column and in 1984 received an award from the Royal Dutch Publishers League. His writing was always distinguished by its clarity and demonstrated his belief that one should beware the notion that the use of long words is a mark of scholarship.

¹⁹ That is to distinguish between the concepts that a person has acquired within society, and those of which mathematics educators have written.

²⁰ Later, Freudenthal was to supervise the Ph.D. work of Adrian Treffers (1978) and Fred Goffree (1979).



I C M E
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RL

Regular Lecture

European Economic Co-operation, later to become OECD: Organisation for Economic Co-operation and Development) seminar held at Royaumont in 1959 which helped launch the “modern mathematics” movement. He thought that too many of the participants were mathematicians who knew very little of mathematics education: people whom he later described as setting forth their ideas on the curriculum in a manner equivalent to supplying “theorems without proof” (Freudenthal, 1963). Whether or not he could have stemmed the tide of “modern mathematics” that flowed from that meeting is doubtful, but, at least, his voice of dissent would have been reported in the influential proceedings. The vital argument, as he saw it, is set out in the title of a paper he published in 1963 (*op. cit.*): “Teaching of modern mathematics or modern teaching of mathematics?” He, himself, thought that changes in teaching methods were needed more than those in curricula.

I shall come to his time as President of ICMI later. At this point I wish to draw attention to the impact upon the mathematics education community of the publication, in 1973, of his *Mathematics as an Educational Task*. This was a book whose like we had not encountered before and it immediately became the centre of discussion and admiration. There is no time here to consider the contents of this and Freudenthal’s subsequent books²¹ on mathematics education in any detail. Brief accounts can be found in Goffree (1992) and more extended ones in *The Legacy of Hans Freudenthal* (Ed. L. Streefland, Dordrecht, Kluwer, 1993). In these can also be found descriptions of the Institute, IOWO, that Freudenthal founded in 1970, its work and the way in which it gradually evolved to become the Freudenthal Institute of the University of Utrecht²².

Klein and Freudenthal

But what were these two men like as persons? We can only hazard guesses concerning Klein. Certainly, his photographs suggest a solid bourgeois figure with confidence in his own ability and, no doubt, anyone who at the age of 23 ventured to devote his inaugural lecture to setting out a programme for national education was not lacking in confidence. One felt that one would learn much from a discussion or over a meal with Klein, but there would be little laughter on the way. Yet I was to enjoy many happy meals and conversations with Freudenthal. True, I learned much, but what I shall never forget are his eyes and the mischievous glint that appeared in them from time to time as a prelude to his recounting yet another anecdote – often, it must be admitted, of a slightly malicious character. We have all, probably, read or heard of the mathematics educator who “learned his mathematics from Piaget and his psychology from Bourbaki”. But I could recount many more such stories. For example, of the time he took a new book on mathematics education to read on a train journey. “It was so bad that I could read it no longer. I opened the train window and threw it out. Afterwards I was overcome by regret. Not because I was despoiling the countryside, but rather because some poor soul might pick it up, read it and take its contents seriously. I should have taken the book home and burned it.”²³ Yet past sins could be

²¹ 1978 *Weeding and Sowing. Preface to a science of mathematics education*

1983 *Didactical phenomenology of mathematical structures*

1990 *Revisiting mathematics education. China Lectures*

²² It is also important to note the contributions made by Freudenthal to the work of the International Group on the Psychology of Mathematical Education (PME) and the CIEAEM (the International Commission for the Study and Improvement of Mathematics Teaching).

²³ There were times, however, when one wondered whether one was listening to Freudenthal the novelist or Freudenthal the mathematician!

forgiven. I once asked Freudenthal to recommend a possible speaker to me for a public lecture on the history of mathematics. “Try X”, he said, “He is one of the best historians of mathematics around”. “But”, I replied, “five or ten years ago you told me that you had just reviewed a book of his that was full of errors and that he had no real grasp of the subject”. “Yes”, he added, with that twinkle in his eye, “but he has learned a great deal since then”.

However, one should not hide Freudenthal’s many achievements in a collection of amusing anecdotes. In particular, I should draw attention to the way in which he was to present his criticism in print. This he approached in a thoroughly professional way. Whether he was criticising, the First International Mathematics Study, Piaget’s theories on the development of number in children, Bloom’s theories on school learning, or the way in which probability was being introduced to pupils in schools, his criticisms were firmly based on original sources and powerfully stated. In this way Freudenthal set himself apart from many educators of that time who concentrated on selling their own wares and would not set time aside to criticise the work and theories of others. Perhaps, they entertained a “dog does not eat dog” feeling: yet such an approach does not lead to the strengthening of mathematics education.

My first meetings with Freudenthal, apart from attending his talks, came when both of us served on the programme committee for ICME 2, held at Exeter, England, in 1972. Since 1972 marked the centenary of Klein’s appointment at Erlangen it seemed sensible to celebrate that and to ask Freudenthal, a renowned group theorist, to give a plenary talk about Klein’s work. Freudenthal accepted the invitation to speak but then said that he did not wish to talk specifically about Klein, but more generally about groups – for some work on group theory was at that time finding its way into the school curriculum. His talk, “What groups mean in mathematics and what they should mean in mathematical education” (Freudenthal, 1973) is still very well worth reading. It shows clearly Freudenthal’s historical grasp and his ability to supply examples from a wide range of contexts, but most importantly it shows the divergence between his approach to mathematics education and Klein’s. “By stressing groups as a formal means of classifying geometries, Klein forgot about groups as a working tool *within* a geometry. This was the reason why school geometry was never influenced by the *Erlanger Programm*. Not unlike Klein’s book *Elementary Mathematics from an Advanced Standpoint*²⁴, his *Erlanger Programm* hovered too high above school mathematics to be able to influence it.” In his talk, Freudenthal took the opposite approach by considering what should be presented as an elementary view of an advanced topic: a view that would give a correct mathematical impression and not degenerate into the learning of a sparse collection of techniques. “If children are taught groups they are entitled to learn genuine group theory rather than a childish version. In the past mathematics has suffered under the falsifying tendencies in adaptations of

²⁴ Although Freudenthal’s criticism of Klein’s book is not unreasonable, Klein made a remarkable attempt to introduce teachers to a wider conception of the mathematics they were teaching. It was far removed from the “Schoolmaster’s Guides” that had been in circulation for many, many years. Klein also provided some ideas relating to schoolwork that seemed ahead of their time. I recall, particularly, his suggestion for using the old-style Brunsviga (analogue) calculators for helping children learn arithmetic. Unfortunately, cost would have ruled this, otherwise admirable, suggestion out of court. A more recent, similar attempt to view high school algebra and calculus from a “modern” mathematical viewpoint was *A Companion to A-level Mathematics*, Armitage, J.V. and Griffiths, H.B. (Cambridge U.P., 1969). This too, although, well written, had little influence on teachers who tended to see it as too rarefied and having no consequences for their teaching. It is “Advanced mathematics from an elementary point of view” that would seem to carry with it most rewards (educational and financial!).



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RL

Regular Lecture

mathematical subject matter to school level. Let us be more cautious in the future. Honesty is a cardinal virtue in education. Nothing is lost if some subject matter cannot be taught prematurely and much is gained if it can in an honest way.²⁵,

There was then a sense of *de haut en bas* about Klein²⁶ that Freudenthal tried to avoid.

Klein, Freudenthal and ICMI

Now let us turn to ICMI and to the contributions that Klein and Freudenthal made to that.

The first decade of the twentieth century was, in many countries, a most important one for their educational systems and, in particular, for school mathematics curricula. Not surprisingly, then, it was one of the three periods chosen for study at the meeting held in Geneva in 2000 to celebrate the centenary of *L'Enseignement Mathématique*, the first international journal on mathematics education and, since its inception, ICMI's official organ²⁷.

To take a few examples, Germany witnessed the Merano Conference and the establishment of experimental schools to trial new curricula; in France the bifurcation between secondary schools for the humanities and for the sciences was abandoned in 1902 and at the same time the position of mathematics within the overall curriculum was enhanced; England saw the establishment of state secondary education in 1901 and also experienced enormous curriculum activity resulting from the ideas of Perry and the decision of Cambridge University in 1903 to accept proofs and orderings other than Euclid's in its requirements for entry; Japan, the USA and some other European countries were also influenced by what the Germans termed *Perryismus* and this, together with other factors, generated much discussion about curriculum reform.²⁸

²⁵ There are now few, if any, attempts to teach group theory in school, but the points that Freudenthal made still give rise to concerns relating to the early teaching of probability and statistics. Freudenthal, 1972, is an interesting paper on the teaching of probability that illustrates the probable dangers that lie in wait for the teacher who bases his/her teaching of probability on some common practical classroom examples.

²⁶ Schubring (1989) tells how Klein negotiated with the Prussian Ministry of Education to have curriculum changes imposed "from above". The Ministry in an "unexpected and certainly most unusual response" answered that they wished to organize the introduction of changes "from below, by enlisting the support of appropriately trained teachers who would act as agents for the implementation of the reforms in selected schools". This led to the work of the Merano Conference. (Regrettably, other ministries in more recent times have not displayed the sensitivity and good sense of the Prussian Ministry!) Klein's response was to turn his attention from the Ministry to the teachers and, true to their word, the Prussian Ministry supported the movement for curriculum change by nominating five secondary schools as experimental institutions.

²⁷ See Coray, D., *et al.*, 2003.

²⁸ For Germany, see, e.g., Schubring, 1987, 1989, 2003; for France, e.g., Belhoste, 1995, Schubring, 2003 (but note that, as in many other countries, separate streams for arts and science students were to be found in the higher classes of these new, unified schools), for England and *Perryismus*, e.g. Howson, 1982; for Japan, e.g. Fujiwara, 1912. Fujiwara's comments on curriculum reform echo those of the Prussian Ministry: reform best comes from below - "That mere change is not identical with bettering ... is too obvious to be mentioned; and yet, looking back ..., one cannot but be struck by the fact of how obvious a truth has been lost sight of. Many of the reforms which have proved to be real improvements, originated in individual initiatives [rather than by central decree]."



RL

Regular Lecture



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Regular Lecture

The need to exchange information and experiences was strongly felt and this manifested itself in the Education Section of the 1908, Rome, International Congress of Mathematicians. In that, papers were presented on mathematics reform in many countries.²⁹ The idea of going further than this and establishing an international commission to enquire into mathematics education had been proposed by the US teacher-trainer, D.E. Smith, in 1905 in *L'Enseignement Mathématique* (p. 469). As a result the 1908 Congress passed a resolution empowering Klein, Fehr and Sir George Greenhill (UK), the “Central Committee”, to form a commission that would compare methods and syllabuses for mathematics education in the secondary schools of the various countries. This commission, the *Commission Internationale de L'Enseignement Mathématique* (CIEM – ICMI in its anglicised form) had Klein as its President and met for the first time in Cologne in September, 1908.

The composition of the commission was based on the attendance of a country's mathematicians at the last four ICMs. Countries who had sent at least two members to at least two of the four ICMs could be represented on the Commission and could form a national sub-commission. If they had sent at least ten members to the ICMs, then they could have two, or in some cases, three members. These criteria yielded eighteen countries. However, an ICM at that time presented very different problems to would-be members than one today. The journey from Australia or Japan to Europe, where the ICMs were held, would have been measured in weeks rather than hours. No wonder, then, that these two countries and, indeed, all others outside Europe, with the exception of the USA, failed to meet these conditions. As a result invitations to become non-voting members were issued to these two and to thirteen other countries thus ensuring that all continents were represented.³⁰

The Commission had several other meetings before the next ICM (Cambridge, 1912). At some of these meetings general topics were discussed; at others attention was concentrated on the following topics.

Milan, 1911

- (1) Rigour in the teaching of secondary school mathematics
- (2) The integration of the teaching of different branches of mathematics
- (3) The teaching of mathematics to those students studying the physical and natural sciences

Cambridge, 1912

- (4) The mathematical training of the physicist at university
- (5) Intuition and experiment in mathematics teaching in secondary schools

²⁹ E.g. by Gutzner (Germany), Smith (USA), Fehr (Switzerland) and Godfrey (England). Smith's talk (ICM, 1909) presents some questions to which comparative answers might be found. Most of the questions would still merit investigation: the questions remain the same, only the contexts have changed.

³⁰ Schubring, 2003, contains a list of the voting and non-voting member countries and also indicates which of the latter demonstrated “at least temporarily a certain degree of activity”. (It is by no means obvious, however, on what matters votes would have been taken.) Schubring also gives a list of members of ICMI in 1914. The list was heavily over-weighted in favour of university mathematicians (many of whom were well known, e.g. Carlsaw, Hobson, Hadamard, Castelnuovo, Enriques, Sounine, Osgood and J.W. Young), but there were a few with a secondary school or teacher training college background (notably, Smith and Godfrey). There is no indication that the list contained any females.

Paris, 1914

- (6) Results obtained from the introduction of calculus in higher secondary schools
- (7) The mathematical preparation of engineers
- (8) The training of mathematics teachers for secondary schools³¹

It will be observed that not all of these fell within CIEM's original remit which restricted its study to secondary schools. This remit, had, however, been extended by the Central Committee to consider all forms of institutionally-based mathematics education. Yet the topics studied were all close to Klein's heart and many reflected his teaching experiences. One suspects, then, that he played a considerable role in determining these themes. Also, although considerable emphasis was laid on university education, the problems considered were centred on "service" teaching, rather than the teaching of mathematicians. Other notable omissions were any consideration of elementary education or of any psychological principles that might guide mathematics teaching. Yet, outside the Commission, the latter was a matter gaining greater recognition.

The actual reports demonstrate both missed opportunities and also some rather remarkable findings. In the former category one would place that on the integration of mathematical topics. Knowing Klein's background, one would have expected the second study to have had something serious to say about possible links between geometry and algebra, and the comparative virtues of, say, a "pure" approach to conic sections as against one utilising coordinate geometry (part of a debate that had gone on since the eighteenth century). Yet it appears that controversy centred on whether or not it was preferable to integrate the teaching of two-dimensional and three-dimensional geometry.³² On the other hand, I was surprised to learn from Godfrey's report on England (carried out in connection with (5)), that over half the schools that replied to the questionnaire undertook out-of-doors practical work in surveying and mensuration, and taught descriptive geometry (in the sense of Monge). The diagrammatic representation of statistical data was also commonly taught and, somewhat more surprisingly, half the schools claimed to use vectors in the teaching of mechanics and/or complex numbers (Howson, 1982). For the mathematicians, the most cheering news to emerge from the reports arose in connection with (7). Lietzmann, Klein's great helper, was able to report that "the engineers want ... to get their mathematics from the mathematician, not from the engineer" (Schubring, 1989, p.190)

CIEM, then, was dealing with important questions, which still concern us today: but the view of mathematics education displayed in its subjects for study was an extremely limited one, and one that was dominated by the interests of mathematicians rather than by those of students at large.

Yet, the main work of CIEM lay elsewhere: the preparation of a vast report on teaching practices in the member countries. Each country was asked to appoint a sub-

³¹ Reports on (1) – (3) can be found in *L'Enseignement Mathématique* **13** (1911) and on (6) and (7) in **16** (1914): those on (4) and (5) in *Proc. 5th ICM (1912)*, Cambridge University Press, 1913, vol. 2. The last study was delayed by the war and a report was not presented until the 1932 ICM. Summaries of the reports reprinted in *L'Enseignement Mathématique* can be found in Schubring, 2003.

³² Fujiwara (1912) in a report written for CIEM raises extremely strong objections to the new Japanese syllabus, prepared by men from the "so-called 'pedagogic school'", whose "characteristic trait is the fusion or mixing up of arithmetic, algebra and geometry". Japan, in his view, had taken the idea of fusion at least one step too far!



RL

Regular Lecture



commission to prepare a national report and the result was outstanding in terms of both quality and quantity, for often a country's report ran to many volumes³³. Nothing on the same scale had been attempted before, or has been attempted since. Regrettably, though, perhaps because of the 1914-18 war, few attempts were made to analyse and draw conclusions and inferences from the vast amount of data that had been generated.

CIEM, however, was not established as a permanent body, but as a comparative study with a limited life and so was formally dissolved in 1920.³⁴ It was reconstituted in 1928 and continued in a lacklustre fashion until the outbreak of war in 1939³⁵. Only in 1952 was it reconstituted and attained a permanent existence as ICMI.

RL

Regular Lecture

What then did Klein contribute to ICMI? First, there can be no doubt that the original Commission accomplished much in its six active years of existence. Klein must take much credit for this by not only providing it with sound advice and guidance, but perhaps more importantly, and as a result of his importance throughout the mathematical world, by giving it professional credence. With Klein at its head, this was clearly a commission to which mathematicians were happy to belong and to whose work they wished to contribute.

But what of the ordinary schoolteacher? What would they know of CIEM's work? Here, I can only give an impression based upon England. Many schools would have received questionnaires relating to some of the studied themes, and the two volumes that Britain contributed to the study (and which were published by our equivalent of a Ministry of Education) included papers by schoolteachers and would have been widely read throughout the country. The *Mathematical Gazette*, a periodical for teachers, carried regular reports of the CIEM meetings and studies, and a small number of schools would even have welcomed as a guest, Georg Wolff, who as part of the German CIEM contribution wrote an outstanding account of English secondary mathematics education viewed through German eyes³⁶. All in all, I suspect that the average English secondary school teacher at that time was more aware of the activities of the CIEM than today's teachers are of those of ICMI.

Let us now fast-forward the story of ICMI to the 1950s, shortly after its reconstitution. My first contact with ICMI came at the International Congress of Mathematicians held in Edinburgh in 1958. It was a meeting that I attended as a mathematician rather than as an educator. However, finding many of the mathematics talks too specialised for my more general interests, I sampled the meetings of a body whose existence was previously unknown to me. My hazy recollections are of meetings of forty or fifty people who, first and foremost, were grateful that they had arrived somewhere where they could comprehend the speaker for more than the first ten minutes. Let us,

³³ The official list of publications issued in 1920 lists 294 contributions published in 17 countries (Schubring, 2003). Although seriously ill in the years 1911-1912, Klein devoted considerable energy to the organisation of the German reports.

³⁴ See Coray *et al.*, 2003, p. 91 for a slightly more extended account of the Commission's history.

³⁵ A fuller account of ICMI's activities over the years up to 1980 can be found in Howson (1984).

³⁶ Wolff, G., *Der Mathematische Unterricht der Höheren Knabenschulen Englands*, Leipzig, Teubner, 1915. Wolff lived to attend ICMEs 2 and 3.



I C M E
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RL

Regular Lecture

however, take a more official view³⁷ of the position, for it is important to understand what Freudenthal set out to change.

ICMI was allowed a total of seven and a half hours of the eighth day of the Congress in which to organise its sub-pleinary activities. This it divided up into three morning and three afternoon sessions devoted to talks and discussions on topics previously proposed for the Congress and on which national sub-commissions had been asked to submit reports. The topics in 1958 were:

- mathematical instruction up to the age of 15;
- the scientific foundation of mathematics at the secondary school level;
- comparative studies of teaching methods for introducing geometry (for which Freudenthal acted as rapporteur³⁸).

The rapporteur was allowed one hour, as were the national sub-commissions. The final half-hour was for general discussion.

I shall not dwell on the ridiculousness of trying to deal in any sensible way in two and a half hours with, say, mathematical instruction up to the age of 15 in the then 23 ICMI countries having national sub-commissions.

More interestingly, ICMI had earlier received a proposition from Heinz Hopf, at that time President of the International Mathematical Union (and previously Freudenthal's PhD supervisor), that it should conduct a study into the difficulties of recruiting mathematics teachers in the various ICMI countries. This gave rise to ICMI sending out a questionnaire on the topic to its sub-commissions (*L'Enseignement Mathématique* (2)4 (1958)).³⁹ However, I have been unable to trace any report on the outcome of this enquiry; certainly it is not referred to in the report of ICMI for the period 1959-62.

Much was to happen in mathematics education before the next ICM was held in Stockholm in 1962. This was a time when UNESCO and OEEC (the predecessor of OECD) took a considerable interest in mathematics education and, moreover, had funds available to finance initiatives in that field. ICMI's advice was sought, therefore, in connection with OEEC's seminar at Royaumont and the subsequent meeting in Dubrovnik which resulted in the publication of *Synopses for Modern Secondary School Mathematics* (OEEC, 1961). New curricular projects were springing up in many countries and it was as the newly appointed editor of one of these, the School Mathematics Project, that I attended the Stockholm ICM. Again, the limited time given over to ICMI was devoted to three themes⁴⁰:

³⁷ Based on circular letters sent to all ICMI national sub-commissions to be found in *L'Enseignement Mathématique*, (2)3 (1957) 76-9; 300-306 and a report of the Congress in (2)4 (1958) 224-5.

³⁸ Freudenthal (1963) was later to comment on the lack of interest in this topic shown by national sub-commissions. A report on the outcomes is to be found in *L'Enseignement Mathématique*, (2)5 (1959).

³⁹ This problem, which arose in the majority of the sixty-two countries surveyed, had been set out by UNESCO in its report *L'Enseignement des mathématiques dans les écoles secondaires*.

⁴⁰ The mystic number three, continued to dominate ICMI's thinking concerning its contributions to the ICM. In 1966 (Moscow) the three themes were: the role of the problem in the development of mathematical activity; the use of the axiomatic method in secondary school teaching; and whether or not a special course was needed in universities for the mathematical training of physicists. Interestingly, this led to my first actual involvement in ICMI work (other than in helping to form a respectable quorum at ICM meetings). The Association of Teachers of

modern mathematics in the secondary school; the teaching of algebra and arithmetic⁴¹; the training of teachers.

One learned little from such sessions and unfortunately few of the growing number of those involved full time in work on mathematical education felt it worthwhile to visit Stockholm for such a meagre official offering.

By this time, however, it was becoming clear that ICMI in its then form was incapable of meeting obvious needs. Some countries withdrew from ICMI and others demonstrated a clear lack of interest. This disinterest was closely related to the shortage of finance that prevented ICMI from doing what it thought was desirable. An attempt to raise more funds was made in 1959 and member countries were asked to make specific contributions to its work in multiples of a unit contribution of \$25.⁴² The special appeal resulted in the following contributions:

1959-62: a total of \$350 from Denmark, Italy, Luxembourg, the Netherlands and Sweden, plus a more substantial donation of \$1250 from the USA.

1963-65: a total of \$350 from Sweden, the Netherlands, Luxembourg and Italy.

One will not ask questions concerning France, Germany, the UK, but what was given would not have gone far. Was this, however, a judgement on the perceived efficiency of ICMI and the way in which it was operating?

In its report for 1959-62, the Executive Committee set out what it thought ICMI should aim at doing – provided of course that finances were available:

- support at least three scientific meetings per year, at least one of which should be outside Europe;
- consider how it might satisfy the growing demand for information concerning studies, experiments and action within mathematics education;
- extend its activity to new areas, such as Africa.

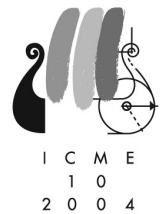
In respect to the last aim, it is worth recalling that in 1961, ICMI, with considerable financial support from UNESCO and other bodies, had organised an Inter-American Conference on Mathematical Education (IACME) in Bogotá (Columbia)⁴³. A further important step towards realising the third aim came in 1964, when it was agreed that a country could become attached to ICMI and form a national sub-commission even if it was not a member of IMU. Thus it was recognised that ICMI, serving the needs of mathematics education, had a far wider target population than did its parent body, the IMU, which concentrated on the needs of those involved in mathematics research.

Mathematics was prompted to write a booklet in connection with the first theme (*What is a problem?*) and I well remember taking part in the workshop that preceded its writing and learning much from that participation. This demonstrated to me the value of such “theme” work, if national sub-commissions or teacher associations proved sufficiently interested in them to set up working groups and to produce reports that would arouse interest amongst, and prove of benefit to, teachers in their own country and elsewhere.

⁴¹ Freudenthal (1963) complained that everyone now wanted to talk and write about geometry!

⁴² ICMI’s annual subvention from IMU (and which came out of the member countries contributions to that body) was currently \$300 per year. Some other money, for the organisation of scientific meetings came to ICMI via IMU from ICSU, the International Council of Scientific Unions. (Lehto, 1998)

⁴³ The IACME meeting cost \$36,500, which puts into contemporary context the amounts ICMI received from its special appeal.



RL

Regular Lecture

Senegal was the first African country to become attached to ICMI and set a path which many others were to follow.

Finally, before coming to the Freudenthal years, we note that the Executive Committee's report for 1962-66 (*L'Enseignement Mathématique*, (2)12 (1966), pp. 131-8) recommended that ICMI should seek to establish a permanent Secretariat⁴⁴ led by a competent scientist who would devote the major part of his time to its direction. A call frequently repeated since, but to no avail.

When Freudenthal assumed the Presidency of ICMI in January 1967, he was faced with two alternatives: he could carry on as usual or he could try to break what was by then becoming the ICMI mould. He chose the latter. How was he to take his Executive Committee and IMU with him? In the words of a member of the former: "Freudenthal was an autocrat⁴⁵ – for better or worse. I always felt that he did what he liked with little reference to anyone else. No bad thing, maybe ...". IMU had similar feelings: "at the meeting of the IMU Executive Committee held in Paris in May 1968, President Cartan and Secretary Frostman complained of the lack of information about the activities of ICMI. The Executive Committee had not been told of the creation by ICMI of the new journal *Educational Studies in Mathematics*, ... A financial contract had been signed between ICMI and UNESCO without the IMU having been informed. And ... it seems that ICMI decided to hold an international congress in Paris in 1969". (Lehto, 1998, p. 259)

Autocrat or not, Freudenthal got things done and, with financial aid from the French Government and UNESCO, the first International Congress on Mathematical Education was held in Lyon (rather than, as initially reported, in Paris) from August 24th to 30th, 1969. That first ICME should not be taken as a model. The great contribution of Klein and Freudenthal to ICMI was to establish precedents that could be developed – foundations on which to build. The Congress was programmed to take place within one large congress hall. Mornings were devoted to twenty-one plenary talks and the weekday afternoons (bar the one on which a memorable excursion to the Beaujolais vineyards took place) to a series of 15-minute talks. The latter were distinctly depressing, with an ever-changing audience but with rarely more than fifty members, coming to support their friends or to escape the somewhat unseasonable weather. Complaints from members led to one or two evening discussions being added to the programme in the latter part of the week.⁴⁶ A hint as to what might be encouraged at later ICMEs was given by the workshop organised by the UK Association of Teachers of Mathematics which took a class of children to Lyon and, as part of the workshop, conducted demonstration classes which provoked considerable discussion

⁴⁴ In the past there was sometimes opposition to this idea on the grounds that it could condemn the President and the Secretary-General to being in different continents. However, the coming of e-mail, telephone and video conferences no longer make this an obstacle.

⁴⁵ Hanna and Sidoli (2002, p.126) tell how Freudenthal chose the seventeen members of the *Educational Studies in Mathematics* Editorial Board himself, but then never used them for refereeing purposes: "he would send [them] a note saying 'I intend to publish the following articles in the next issue. Please let me know if you would like to read any of them'".

⁴⁶ One of these will, I fear, never be erased from my memory. Against my initial better judgement, I agreed to chair a discussion on one of ICMI's perennial topics: the place of axiomatics in school mathematics. What I had failed to take into account was (i) the enormous amount of heat that had just been generated in France by the introduction of a new, extremely abstract mathematics curriculum, and (ii) just how quickly the French can speak their language when overheated. I have rarely been present at such an ineffectually chaired meeting!



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Regular Lecture

and comment: “It transmitted an image and philosophy in a way that could not have been achieved by any number of plenary lectures” (Howson, 1973, p. 5)

Freudenthal, then, did not stray far from the organisational pattern of an ICM. Yet mathematics and mathematics education are very different in their natures.⁴⁷ What was welcome was the number of women plenary speakers: Zofia Krygowska, G.G. Maslova, Frédérique Papy and Emma Castelnuovo⁴⁸. What to me was less so, since at that time I was employed full-time on educational work in Africa, Asia and the Central and Southern Americas, was that the speakers came entirely from Europe and North America and their talks concentrated on the problems of schools in their own countries. Also, unlike Klein, Freudenthal paid no attention when selecting speakers to the problems of teaching mathematics in universities or, indeed, in any educational institutions other than primary and secondary schools.⁴⁹

What of Freudenthal’s other outstanding innovation? The provision of satisfactory vehicles for the dissemination of information was regularly stressed in the reports of the retiring Presidents of ICMI and relations with *L’Enseignement Mathématique* were a constant topic of discussion. At a colloquium held in Utrecht in 1964, members openly questioned the continued appropriateness of the relationship with *L’Enseigne-*

⁴⁷ My views on this were spelled out in the Proceedings of ICMI 2 (Howson, 1973, p. 4)

⁴⁸ Zofia Krygowska had been the rapporteur for the “problem” theme at the Moscow ICM in 1966. This was, I believe, the first time a woman had been invited to carry out an important role at an ICMI meeting. The first woman member of the ICMI Executive Committee was Anna Sierpiska (1991-4). Emma Castelnuovo provided an interesting link with ICMI’s beginnings, for her father, Guido, and her uncle, Federigo Enriques, were both members of the original Commission.

⁴⁹ Another significant omission, as at ICME 2, was any account of developments within mathematics itself. Writing in the Proceedings of ICME 2, I recalled how “many of those who ... attended ICM congresses primarily for the work of the Education Section also greatly enjoyed the opportunity to make contact with leading research mathematicians and to learn of the most recent developments. It must be admitted, however, that the rapid expansion of mathematics has tended to diminish the value of such contacts, for now a vast amount of specialist knowledge is required before one can comprehend the significance of developments - particularly when they are presented in a manner which assumes specialist competence in the listener. The idea of attempting to bridge this gap by means of a series of expository lectures intended for a general audience was discussed by the [Programme] committee. In the event, it was unable to mount such a programme – but it remains the hope of the committee that this idea will be re-examined and followed-up at future ICMI congresses” (Howson, 1973, p. 12). This was, in fact, done, with success at Berkeley (ICME 4), but not, so far as I can recall, since.

The Exeter programme was to have included one plenary lecture on a mathematical topic, for René Thom was invited to give an introductory lecture on catastrophe theory. He asked, however, that he might be allowed to speak on the educational topic of modern mathematics and its teaching. The committee at first were unhappy about this, but Freudenthal (who was well aware that Thom shared his opinions) persuaded us to accept Thom’s suggestion. The resulting talk, “Modern mathematics: does it exist”, proved, without doubt, the most quoted lecture ever given at an ICME. Another omission at Lyon which was partially remedied at Exeter, but which has persisted at most other ICMI, was the absence of invited speakers from outside mathematics and mathematics education. The absence of non-academic members from the Executive Committee of ICMI (with the notable exception of Henry Pollak) is, perhaps, understandable, but it is strange that the mathematical education community feels that it has little to gain from listening to those who use mathematics, whose employees use mathematics, who ask mathematicians to provide service teaching in their departments, who administer schools and institutions in which the teaching of mathematics has an important role, or who are specialists in such fields as psychology, sociology,



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Regular Lecture



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Regular Lecture

ment *Mathématique*, which was now felt to be of more interest to mathematicians than mathematics educators. Moreover, a true ICMI journal should accommodate more languages and be more widely distributed. A resolution was passed (see Freudenthal, 1978, pp i-ii) that: “The participants ... feel the urgent need for more international information on national activities in mathematics education, which could be organized and spread by an active and accessible centre of information or by a high level periodical on mathematical education”. While we still await the establishment of such a centre/secretariat, Freudenthal, as the minutes of the IMU made clear did establish what was, at some stage, intended to be an ICMI journal.

Quite how *Educational Studies in Mathematics* came to be divorced from ICMI is still a mystery that I have been unable to solve. Certainly, the IMU expressed worries about the financial implications of both of Freudenthal’s two major initiatives. This might have persuaded him to “go it alone” so far as *ESM* was concerned. However, my suspicion is that he realised that establishing *ESM* as an ICMI journal meant handing over control of it at the end of his Presidency. This meant more than just losing control of his “baby”; it meant condemning it to an unknown future with changing Executive Committees that still lacked the fixed central base needed to give ICMI its desired stability⁵⁰. As Lehto remarks in his history of the IMU, (Lehto, 1998, p. 258), “For a long time ICMI’s activities visibly suffered from a lack of adequate administration. This was the case, for example, in the years 1979-1982.”

I feel, then, that *ESM* gained from the “divorce”. It is now the “high level” periodical that the Utrecht Colloquium sought. However, it does not entirely serve the needs identified by that colloquium. Certainly, it satisfies the needs of researchers in mathematics education⁵¹. However, the need for disseminating information on activities within mathematics education still exists.

The international exchange of information was to some extent met in the 1960s and ‘70s by UNESCO’s occasional publication, *New Trends in Mathematics Teaching*, for which ICMI took editorial responsibility. Thus, for example, Volume 3 (1973) was based on an ICMI seminar held in 1971, and presided over by Freudenthal; Volume 4 (1978) on the findings of the 13 working groups at ICME 3 (Karlsruhe, 1976). Thanks to generous funding from UNESCO, whose mathematics officer at that time was Bent Christiansen, who is still remembered with great affection - particularly here in Copenhagen - the work of Karlsruhe, led by Hans-Georg Steiner, was the best prepared of any ICME. Again, Freudenthal played a role in its planning. *ESM* also played its part in that decade, in particular, by devoting two special issues to reports from 16 countries on the changes that had taken place in them in school mathematics post-1950: the aims of the changes and the extent to which these aims were realised⁵².

Nowadays, however, at a time when governments spend vast sums of money supporting the activities of PISA and TIMSS and then reacting to their countries’ position in the resulting “horse race”, very little appears to be being done at an international level to disseminate information concerning activities and ideas aimed at improving mathematics teaching, providing more appropriate curricula, making

⁵⁰ At the meeting of ICMI held at the Nice ICM in 1970 and presided over by Freudenthal, there were specific criticisms by Freudenthal and others concerning the manner in which ICMI committee members were chosen. (*L’Enseignement Mathématique*, (2) 16 (1970), p. 198).

⁵¹ Hanna and Sidoli (2002) analyse the contents of the first fifty volumes of *ESM* and it is interesting to note changes of emphasis in the contributions and the contributors. Their comments in Section 3, “Increased emphasis on research papers”, are particularly relevant.

⁵² *ESM* 9 (1978), pp. 143-504



teacher training and the continuing professional development of teachers more effective, and persuading more students to continue with the study of mathematics and more and better-qualified teachers to enter the profession; to draw attention to significant research findings⁵³, etc. This is an omission that ICMI has wished to fill for many decades now. Is the time ripe for a concentrated effort to obtain funding for the establishment of a secretariat that, as one of its duties, would produce yearbooks describing noteworthy activities throughout the world?⁵⁴

Certainly, we can be sure that were Klein and Freudenthal still with us then they would be searching for new ways in which to increase ICMI's influence and usefulness. For the message that must still be emphasised is that these two great servants of ICMI did not provide us with traditions to be followed, but with foundations on which to build.

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⁵³ Hanna (2003) tells us that there are now over 300 journals devoted to mathematics education. Not all of these are concerned with research in mathematics education, yet it would seem valuable for countries to draw international attention to any research that has taken place in their country that could be of international significance.

⁵⁴ Such a move might also help strengthen ICMI in another way. Under Klein, the national members of the Commission played an important role – they were the countries' representatives on the Commission and could attend its meetings. That not all of them actively contributed to the Commission's work can be inferred from Schubring (2003). In more recent years, the role of the ICMI representative has diminished. Until 1969, the national representative's major role was to organise his/her country's response to the four-yearly requests for contributions on one or more of the three themes chosen for discussion at the ICM. Again, not all national representatives responded to these requests. With the coming of the ICMEs and the replacement of "ICM themes" by ICMI studies, ICMI has dealt more directly with individuals than through ICMI representatives. (Contact between ICMI and its representatives has also been beset with difficulties. When Secretary of ICMI, I wrote for some years to one European national representative without obtaining any response from him; only by chance did I discover that he had been dead for over ten years.) Yet an international organisation should have active national sub-commissions, and ICMI representatives should be given some responsibilities and credit for carrying these out. The duty of arranging for the writing of the country's contribution to a yearbook could provide that responsibility – and doing it well, its reward.

It is important, however, that such yearbooks should not be priced beyond the reach of impoverished libraries or readers. This means that either subsidies for publication should be sought, or thought given to producing the yearbooks in an unconventional but cheap form, e.g. as CDs.

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Regular Lecture



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Regular Lecture

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