

Promoting equity in mathematics classrooms – important teaching practices and their impact on student learning¹

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Introduction

During the last five years I have been conducting a longitudinal study in the US in which a team of graduate students³ and I have monitored approximately 700 students through four years of three different high schools. The aim of the study was to monitor the impact of different teaching approaches upon students' understanding of mathematics and their relationships with the discipline (Boaler, 2002a), following up on a study of successful and equitable teaching that I conducted in England (Boaler, 1997, 2002b). The study has produced a number of interesting findings, the most significant being the incredible success of one of the schools. At Railside school, an urban school in which the population of students was more diverse and of a lower socio-economic status (SES) than the other two schools, students achieved more, enjoyed mathematics more and stayed with mathematics to higher levels. In addition, the achievement differences that were in place between students of different ethnic groups at the beginning of high school were reduced in all cases and disappeared in some. For example, at the beginning of high school white students were achieving at significantly higher levels than Latino students but the achievement levels of the Latino students steadily increased at the school and by the end of the second year there were no significant differences between the achievement of Latino and white students. My aim in this paper is to focus upon the teaching practices employed at Railside school, and to consider the reasons why they enabled many more students to be successful than is typically the case in urban American high schools.

Description of the study

The Stanford Mathematics Teaching and Learning Study is a 5-year, longitudinal study funded by the National Science Foundation. The aim of the study is to understand the impact of different teaching approaches upon student learning. The study monitored students in three high schools with the pseudonyms: Greendale, Hilltop and Railside. These three high schools are reasonably similar in terms of their size, and share the characteristic of committed and knowledgeable mathematics teachers. They differ significantly in terms of their location and student demographics. Railside High School, the focus of this analysis, is situated in an urban setting and has a diverse student population with students coming from a variety of ethnic and cultural backgrounds. Hilltop High School is situated in a more rural setting, approximately half of the students are Latino and half white. Greendale High School is situated in a coastal community, with very little ethnic or cultural diversity (almost all students are white).

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School:	Railside	Hilltop	Greendale
Approx school population	1500	1900	1200
Study Demographics	38% Latino/a, 23% African American, 20% White, 16% Asian or Pacific Islanders. 3% other groups.	57% White, 38% Latino/a, 5% other ethnicities.	90% White, 5% Latino/a, 5% other ethnicities.
ELL students	25%	24%	0%
Free/reduced lunch	31%	23%	9%
Parent education, % college graduates	23%	33%	37%

Table 1: Schools and students

The three high schools were chosen because they enabled us to observe and study three different mathematics teaching approaches. Both Greendale and Hilltop schools offered students (and parents) a choice between a traditional sequence of courses, taught using conventional methods of demonstration and practice, and an integrated sequence of courses in which students worked on a more open, applied curriculum called the Interactive Mathematics Program (Alper, Fendel, Fraser, & Resek, 2003), or “IMP.” Students in IMP classes worked in groups and spent much more time discussing mathematics problems than those in the traditional classes. Railside school did not offer a choice and the approach they used was ‘reform’ oriented. The teachers worked collaboratively and they had designed the curriculum themselves, drawing from different ‘reform’ curricula such as the College Preparatory Mathematics Curriculum (Sallee, Kysh, Kasimatis, and Hoey), or “CPM” and IMP. Mathematics was organized into the traditional sequence of classes - algebra followed by geometry, then advanced algebra and so on - but the students worked in groups on longer, non-traditional problems. Classes at Railside were heterogeneous as the school had de-tracked classes some years before. We therefore monitored three approaches in the study - ‘traditional’ and ‘IMP’ (as labeled by the 2 schools) and the Railside approach. However, as only one or two classes of students in Greendale and Hilltop chose the IMP curriculum each year there were insufficient numbers of students to include in our statistical analysis. The main comparison groups of students in the study were therefore approximately 300 students who followed the traditional curriculum and teaching approaches in Greendale and Hilltop schools and approximately 300 students at Railside who were taught using reform oriented curriculum and teaching methods. These two groups of students⁴ provide an interesting contrast as they were taught in very different ways.

⁴ In the remainder of this paper we combine the students from Greendale and Hilltop that followed the traditional curriculum.



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Research methods

In order to monitor and analyze the teaching practices in the three schools, we observed hundreds of hours of lessons, many of which were videotaped. These lessons were analyzed in three different ways, details of which are given in a longer version of this paper (Boaler, 2004). These included qualitative analyses, quantitative coding of time spent in lessons (every 30-second period of time in 55 hours of first level classes was coded, yielding 6,800 coded segments) and a detailed analysis of the questions teachers asked students (see Boaler & Brodie, 2004). In addition to our analyses of the teaching practices inside classrooms, we interviewed students in every year of the study to consider their reported experiences and interpretations of mathematics class. Students were typically interviewed in same sex pairs and we interviewed approximately sixty students each year, sampling high and low achievers from each approach in every school taking care to interview students from different cultural and ethnic groups. In addition to interviews, we also administered questionnaires to all the students in the focus cohorts in years 1, 2 and 3 of the study (when most students were required to take mathematics). The observations, interviews and questionnaires combined to give us information on the teaching and learning practices in the different approaches and students' responses to them. In addition to monitoring the students' experiences of the mathematics teaching and learning we also assessed their understanding in a range of different ways. At the beginning and end of Year 1, the beginning and end of Year 2 and the end of Year 3 we administered tests that were carefully written by the team and considered by the teachers in each approach to make sure they fairly assessed each approach. In addition to these tests that matched, as closely as possible, the work students had met in the different approaches, we also designed and administered longer more applied problems that students were given to work on in groups. We also analyzed state tests and additional details of these are given in Boaler (2004). In this paper I will report upon the students' achievement on the tests administered in the first two years of the study as well as their participation in higher-level mathematics classes.

The different teaching approaches

Students in traditional algebra classes at the two schools were taught using a conventional approach – they sat individually, the teachers presented new mathematical methods through lectures and the students worked through short, closed problems. Our coding of lessons showed that approximately 21% of the time in algebra classes was spent with teachers talking to the students, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole class format. Approximately 48% of the time students were practicing methods in their books, working individually, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics problem was 2.5 minutes. Our focused analyses of the types of questions teachers asked, conducted with two of the teachers of traditional classes, showed that the vast majority of their questions were procedural (Boaler & Brodie, 2004). We classified teachers' questions into seven categories, from 325 minutes of teaching. This showed that 97% and 99% of the two teachers' questions in traditional algebra classes fell into the procedural category.

In the teaching approach at Railside, which will be considered in more depth below, the teachers posed longer, conceptual problems and combined student presentations with teacher questioning. Teachers rarely lectured and students were taught in heterogeneous groups. Our coding of time spent in classrooms showed that teachers lectured to classes for approximately 4% of the time. Approximately 9% of



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the time teachers questioned students in a whole class format. Approximately 72% of the time the students worked in groups while teachers circulated the room asking the students questions of their work, and students presented work for approximately 9% of the time. The average time spent on each mathematics problem was 5.7 minutes. Our focused analysis of the types of questions teachers asked, conducted with one of the Railside teachers over 172 minutes of teaching, showed that she asked many more varied questions than the teachers of traditional classes. For example, 61% were procedural, 21% conceptual, 9% probing, and 8% fell into other questioning categories (Boaler & Brodie, 2004). The broad range of questions she asked, although only derived from analysis of one teacher, was typical of the different teachers at Railside who deliberately and carefully shared their teaching approaches, a practice which included sharing good questions to ask students, as will be described below.

Achievement results

At the beginning of high school, we gave all students who were starting algebra classes in the three schools a test of middle school mathematics. Comparisons of means indicated that the students at Railside were achieving at significantly lower levels than students at the two other schools starting the traditional approach ($t = -9.141$, $p < 0.001$, $n = 658$), as can be seen in table 2. The relatively low performance of the Railside students is not atypical for students in urban, low-income communities. At the end of year one we gave all students a test of algebra to measure what students had learned over the year. The test was designed to assess only algebraic topics that the students had encountered in common across the different approaches, and we used an equal amount of question-types from each of the three approaches. The test was designed in conjunction with teachers from each approach in each school. Comparisons of means showed no significant differences between the scores of students in the two approaches – the students in traditional classes were still scoring at higher levels but the differences were not significant ($t = -2.04$, $p = 0.04$, $n = 637$ – see table 2). Thus the Railside students were able to achieve at comparable levels after a year of algebra teaching, despite starting the course at significantly lower achievement levels. At the end of year 2 we gave students⁵ a test of algebra and geometry, reflecting the content the students had been taught over the first two years of school. Again the test was designed by selecting equal numbers of questions from each approach, in conjunction with the teachers at the schools. By the end of year 2 Railside students were significantly outperforming the students in the traditional approach ($t = -8.304$, $p < 0.001$, $n = 512$) as can be seen in table 2. There are fewer students in our geometry classes in Railside only because the timetable was structured so that students could take geometry in second, third or fourth years whereas students in the other schools needed to take it in year two of the study (as will be described in the next section). The students in geometry classes at Railside did not represent a selective group; they were of the same range as the students entering year 1.

⁵ The test was given to all students in geometry classes. These were not the exact same students as those in year 1 as some students chose to take geometry in a later year and some older students joined the classes, who had not been in our algebra sample. But we also performed tests that included only those students who were in year 1 and year 2, so the students who took algebra and geometry and who took the pre-test, end of year 1 test and end of year 2 test. This was a smaller number of students but it also showed that the Railside students started at lower levels and ended at significantly higher levels.

	Traditional			Railside			T-value Sign. Level
	Mean score	Std Deviation	n	Mean score	Std. Deviation	n	
Y1 pre-test	22.23	8.857	311	16.00	8.615	347	-9.141 p<0.001
Y1 Post- test	23.90	10.327	293	22.06	12.474	344	-2.040 p=0.04
Y2 Post- test	18.34	10.610	313	26.47	11.085	199	-8.309 p<0.001

Table 2: Assessment results

In addition to the high achievement of the students at Railside, the students also enjoyed mathematics more. In questionnaires given to the students each year the Railside students were always significantly more positive. For example, in the year 3 questionnaire students were asked to finish the statement: 'I enjoy math in school' with one of four time options: 'all of the time', 'most of the time', 'some of the time', or 'none of the time'. Twenty-nine per cent of students in traditional classes (n=318) chose all or some of the time, compared with fifty-four per cent of students from Railside (n=198) which is a significant difference ($t = 4.758$; $df = 286$; $p < 0.001$). In addition, significantly more Railside students agreed or strongly agreed with the statement 'I like math', with 74% of Railside students responding positively, compared with 54% of students in traditional classes ($t = -4.414$ $df=220.77$; $p < 0.001$). Other years produced similar results: 71% of Railside students in year 2 classes (n =198) for example reported enjoying 'math class' compared with 46% of students in traditional classes (n=318) ($t = -4.934$; $df=444.62$; $p < 0.001$).

By the end of year 4, 41% of Railside seniors were enrolled in advanced classes of pre-calculus and calculus, compared with 30% of seniors at Hilltop and 23% of seniors at Greendale⁶. There were no gender differences in performance in any of the tests we gave students at any level, and young women were well represented in higher mathematics classes. They made up 50% of students in advanced classes at Hilltop, 48% at Greendale and 59% at Railside. The students at Railside school enjoyed mathematics more than students taught more traditionally, they achieved at higher levels, and achievement differences between students of different ethnic and cultural groups were lower than those at the other schools. Our detailed and varied research methods conducted over four years gave us important insights into the reasons for the teachers' and students' success, which I will consider now.

⁶ This percentage includes all seniors at Greendale and Hilltop, whether they attended the 'traditional' or IMP classes. At this time we have been unable to separate the students from IMP but as they were few in number this would not affect the reported percentage greatly.

Analyzing the sources of success

Part 1. The department, curriculum and timetable

Railside school has an unusual mathematics department. Twelve of the thirteen teachers work collaboratively, spending vast amounts of time designing curriculum, discussing teaching decisions and actions, and generally improving their practice through the sharing of ideas. A study conducted by Horn on the ways in which the department collaborate, found that the teachers spent around 650 minutes a week planning, individually and collectively (their paid work week provides 450 minutes of preparation time) (Horn, 2002). Unusually for the United States, the mathematics department strongly influences the recruitment and hiring of teachers, enabling the department to maintain a core of teachers with shared philosophies and goals. The teachers share a strong commitment to the advancement of equity. The department has spent many years working out a coherent curriculum and teaching approach, and they strive to ensure that good teaching practices are shared. One way in which this is achieved is through a practice that the department calls “following.” The co-chairs structure teaching schedules so that new teachers can stay a day or two behind a more experienced teacher, allowing the new teachers to observe lessons and activities during their daily preparation period before they try to adapt it for their classrooms (Horn, 2002, 2005). In addition teachers share teaching practices and moves in weekly meetings in order that students experience a consistent approach.

The mathematics department has focused in particular upon the introductory algebra curriculum that all students take when they start the school. The algebra course is designed around key concepts with questions from various published curriculum such as CPM, IMP and a textbook of activities that use algebra Lab Gear (Picciotto & Wah, 1994). A theme of the algebra and subsequent courses is multiple representations, and students are frequently asked to represent their ideas in different ways, using words, graphs, tables and symbols. In addition, connections between algebra and geometry are emphasized even though the two areas are taught in separate courses. Railside follows a practice of ‘block scheduling’ and lessons are 90 minutes long, with courses taking place over half a school year, rather than a full academic year. In addition, the introductory algebra curriculum that is generally taught in one course in US high schools, is taught in the equivalent of two courses at Railside. The teachers have spread the introductory content over a longer period of time partly to ensure that the foundational mathematical ideas are taught carefully with depth and partly to ensure that particular norms – both social and socio-mathematical (Yackel & Cobb, 1996) – are carefully established. The fact that mathematics courses are only half a year long at Railside may appear unimportant but in fact this organizational decision has a profound impact upon the students’ opportunities to take higher-level mathematics courses. In most North American high schools, mathematics classes are one year long and they begin with algebra. This means that students cannot take calculus unless they are advanced, as the typical sequence of courses is algebra, geometry, advanced algebra then pre-calculus. If a student fails a course at any time s(he) is knocked out of that sequence and has to retake the course, further limiting the level of content they will reach. At Railside the students could take two mathematics classes each year. This meant that students could fail classes, start at lower levels, and/or choose not to take mathematics in a particular term and still reach calculus. This relatively simple scheduling decision is part of the reason that significantly more students at Railside took advanced levels classes at school than students in the other two schools.



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Another important difference between the classes in the three schools we studied was the heterogeneous nature of Railside classes. Whereas incoming students in Greendale and Hilltop could enter geometry or could be placed in a remedial class, such as ‘math A’ or ‘business math’, all students at Railside entered algebra classes. The department is deeply committed to the practice of mixed ability teaching and to giving all students equal opportunities for advancement.

Recent years have pointed to the importance of school and district contexts in the support of teaching reforms. Such support is undoubtedly important but Railside is not a case of a district or school that initiated or mandated reforms. The reforms put in place by the mathematics department were in line with other school reforms but they were driven by the passion and commitment of the mathematics teachers in the department. The school, in many ways, provided a demanding context for the reforms, not least because they had been managed by five different principals in the six years we were there, and they had been labeled an ‘under-performing’ school by the state because of low state test scores. Neither of these factors are atypical in less well resourced school districts in the US. The department fought to maintain their practices at various times and worked hard to garner the support of the district and school, and while the teachers felt well supported at the end of our study Railside does not represent a case of a reforming district encouraging a department to engage in new practices. Rather, Railside is a case of an unusual, committed and hard working department that continues to grow in strength through its teacher collaborations and work.

Part II. Groupwork and ‘Complex Instruction’

Some mathematics departments in the US employ group work with limited success, particularly because groups do not always function well, with some students doing more of the work than others, and some students being excluded or choosing to opt out. At Railside the teachers employed additional strategies to make group work successful. They adopted an approach called ‘complex instruction’ designed, by Liz Cohen and Rachel Lotan in the US (Cohen, 1994; Cohen & Lotan, 1997) for use in all subject areas. The system is designed to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group *interactions*. The approach includes a number of recommended practices that the school employs, as I shall now describe:

1. Roles

When students are placed into groups they are also given a particular role to play, such as ‘facilitator’, ‘team captain’, ‘recorder/reporter’ or ‘resource manager’ (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function. At Railside the teachers emphasize the different roles at frequent intervals, stopping, for example, at the start of class to remind ‘facilitators’ to help people check answers or show their work or to ask the group: ‘what did you get for number 1?’. Students change roles at the end of each unit of work. The teachers reinforce the status of the different roles and the important part they play in the mathematical work that is being undertaken. Although I will not write more about the roles in this paper, they contribute to the complex interconnected system that operates in each classroom; a system in which everyone has something important to do and all students learn to rely upon each other. Readers may consult the literature on complex instruction (or visit www.complexinstruction.org.) for more information on the roles.



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2. Multidimensional classrooms

In many mathematics classrooms there is one practice that is valued above all others – that of executing procedures (correctly and quickly). The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, whilst others sink to the bottom with most students knowing where they are in the hierarchy created. Such classrooms are uni-dimensional – the dimensions along which success is presented are singular. A central tenet of the complex instruction approach is what the authors refer to as ‘multiple ability treatment’. This ‘treatment’ is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different ‘abilities’. Teachers should explain to students that ‘no one student will be “good on all these abilities” and that each student will be “good on at least one”’. (Cohen & Lotan, 1977, p 78).

At Railside the teachers create multidimensional classes by valuing many dimensions of mathematical work. This is achieved – in part – by having more open problems that students can solve in different ways. The teachers value different methods and solution paths and this enables more students to contribute ideas and feel valued. But multiple solution paths are not the only contributions that are valued by teachers. When we interviewed the students and asked them ‘what does it take to be successful in mathematics class?’ they offered many different practices such as: asking good questions, rephrasing problems, explaining well, being logical, justifying work, considering answers, and using manipulatives. When we asked students in ‘traditional’ classes what they needed to do in order to be successful they talked in much more narrow ways, usually saying that they needed to concentrate, and pay careful attention.

The multidimensional nature of the classes at Railside was an extremely important part of the increased success of students. Put simply, *when there are many ways to be successful, many more students are successful*. Students are aware of the different practices that are valued and they feel successful because they are able to excel at some of them. Teachers at other schools may not encourage practices outside of procedure execution because they are not needed in state tests, but the fact that teachers at Railside valued a range of practices and more students could be successful in class made students feel more confident and positive about mathematics. This probably enhanced their success on tests even when tests assessed a more narrow range of mathematical work.

The following comments given by students in interviews give a clear indication of the multidimensionality of classes -

Back in middle school the only thing you worked on was your math skills. But here you work socially and you also try to learn to help people and get help. Like you improve on your social skills, math skills and logic skills (Janet, Y1).

With math you have to interact with everybody and talk to them and answer their questions. You can't be just like “oh here's the book, look at the numbers and figure it out”

Int: Why is that different for math?

It's not just one way to do it (...) It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: “why does it work”? (Jasmine, Y1).

These students recognize that helping, interpreting and justifying are critically valued practices. The following student describes another valued dimension:

A math person is a person who knows like, how to do the work and then explain it. Like explaining everything to everyone so they could get it. Or they could explain it the hard way, the easy way or just, like average – so we could all get it. That’s like a math person I think (Jorge, Y1).

Jorge shows that he appreciates something sophisticated – that explanations can vary and that a “math person” can explain in different ways. Given the frequent explanations these students give and hear, it may be unsurprising – but nevertheless important – that he appreciates the distinguishing qualities of different explanations.

One of the practices that I have come to regard as being particularly important in the promotion of equity is justification. At Railside students are required to justify their answers at almost all times. There are many good reasons for this – justification is an intrinsically mathematical practice (RAND, 2002; Martino & Maher, 1999), but this practice also serves an interesting and particular role in the promotion of equity. Many teachers struggle to deal with the wide range of students who attend classes, particularly in introductory classes such as high school algebra, which include students who are motivated with a wealth of prior knowledge as well as those who are less motivated and /or lack basic mathematical knowledge. Teachers want to help all of the students but the gap between the knowledge of lower attaining and higher attaining students can be very difficult to address. At Railside school classes have as wide a gap as any I have seen but the teachers embrace the diversity they encounter and one practice that helps them support the learning of all students is justification. The following two students give some indication of the role of justification in helping different students:

Int: What happens when someone says an answer?

A: We’ll ask how they got it

L: Yeah because we do that a lot in class. (...) Some of the students – it’ll be the students that don’t do their work, that’d be the ones, they’ll be the ones to ask step by step. But a lot of people would probably ask how to approach it. And then if they did something else they would show how they did it. And then you just have a little session! (Ana and Latisha, Y3).

It is noteworthy that these two students do not talk about students who are slow, dumb or stupid, as other students in our study do; they talk only about students ‘that don’t do their work’, a point to which I shall return later in the paper.

The following boy was achieving at lower levels than other students and it is interesting to hear him talk about the ways he was supported by the practices of explanation and justification:

Most of them, they just like know what to do and everything. First you’re like “why you put this?” and then like if I do my work and compare it to theirs. Theirs is like super different ‘cos they know, like what to do. I will be like – let me copy, I will be like “why you did this? And then I’d be like: “I don’t get it why you got that.” And then like, sometimes the answer’s just like, they be like “yeah, he’s right and you’re wrong” But like – why?” (Juan, Y2).



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Juan also differentiates between high and low achievers without referring to such adjectives as ‘smart’ or ‘fast’, instead saying that some students ‘know what to do’. He also makes it very clear that he is helped by the practice of justification and that he feels comfortable pushing other students to go beyond answers and explain ‘why’ their answers are given. At Railside the teachers have carefully prioritized the message that each student has two important responsibilities – both to help someone who asks for help, but also to ask if they need help. Both are important in the pursuit of equity, and justification has emerged as a helpful practice in the learning of a wide range of students, particularly because the act of justification makes mathematical methods more transparent and learnable for different students.

3. Assigning competence

An interesting and subtle approach that is recommended within the complex instruction literature is that of ‘assigning competence’. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group’s attention; asking a student to present an idea; or publicly praising a student’s work in a whole class setting. This practice was one that I could not fully imagine until I saw it enacted. My first awareness of it came about when a quiet Eastern European boy muttered something in a group that was dominated by two happy and excited Latina girls. The teacher who was visiting the table immediately picked up on it saying ‘Good Ivan, that is important’. Later when the girls offered a response to one of the teacher’s questions he said, ‘Oh that is like Ivan’s idea, you’re building on that’. He raised the status of Ivan’s contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. Cohen (1994) recommends that if student feedback is to address status issues, it must be public, intellectual, specific and relevant to the group task (p 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work, and the specific dimension means that students know exactly what the teacher is praising.

The practice of ‘assigning competence’ is subtle and requires highly sensitive teachers. It is a practice that many good teachers employ, to a greater or lesser extent, without necessarily being aware of it or having a name for it.⁷ This practice is also an important part of the teachers’ commitment to equity and to the principle of showing what different students can do in a multifaceted mathematical context.

4. Teaching students to be responsible for each other’s learning

A major part of the equitable results attained at Railside is the serious way in which teachers expect students to be responsible for each other’s learning. Many schools employ group work which, by its nature, brings with it an element of responsibility, but Railside teachers go well beyond this to ensure that students take the responsibility very seriously. One way in which teachers nurture a feeling of responsibility is through the assessment system. Teachers grade the work of a group by, for example, rating the quality of the conversations groups have. The teachers also occasionally give group tests, which take several formats. In one version students work through a test together, but the teachers grade only one of the individual papers and that grade

⁷ I thank Deborah Ball for pointing this out to me.



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stands as the grade for all the students in the group. A third way in which responsibility is encouraged is through a practice of asking one student in a group to answer a follow up question after a group has worked on something. If the student cannot answer the question the teacher will leave the group to have more discussions and return to ask the same student again. In the intervening time it is the group's responsibility to help the student learn the mathematics they need to answer the question. (As part of my presentation at ICME I showed a video in which this practice takes place, described in Boaler, 2004). The teacher move of asking one member of a group to give an answer and an explanation, without help from their group-mates, is a subtle practice that has major implications for the classroom environment. This practice means that students are responsible to everyone in their group. In the following interview extract the students talk about this particular practice and the implications it holds:

Int: Is learning math an individual or a social thing?

G: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.

B: I think both - because individually you have to know the stuff yourself so that you can help others in your group work and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella and Bianca).

The students in the extract above make the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicate an interesting social orientation that becomes instantiated through the mathematics approach, saying that the purpose in knowing individually is not to be better than others but so "you can help others in your group". The four practices I have described so far – those of taking group roles, multidimensionality, assigning competence and encouraging responsibility are all part of the complex instruction approach. I will now review three other practices in which the teachers engage that are also critical to the promotion of equity. These relate to the challenge and expectations provided by the teachers.

Part 3. Challenge and expectations

1. High cognitive demand

The teachers at Railside provide a huge amount of support to students, making themselves available to students after school and in the evenings. In interviews the students talk at length about the support of the teachers. But the teachers also expect a lot from the students and give them complex, mathematically challenging work. Importantly the support that teachers give to students does not serve to reduce the cognitive demand of the work (Stein, Smith, Henningsen & Silver, 2000). The cognitive demand that is expected of all students is higher than at other schools partly because the classes are heterogeneous and no students are precluded from meeting high level content, but teachers also enact a high level of challenge in their interactions with groups and their questioning.

A very important feature of the curriculum that teachers use, that would not be seen in the curriculum materials, is the act of asking follow up questions. The curriculum example analyzed in the longer version of this paper (Boaler, 2004) involves students finding a perimeter of a complex shape made up of algebra tiles. The

perimeter is $10x + 10$ and the teacher asks the follow up question: ‘where is the 10?’. This is complex because the 10 does not exist in a particular place in the diagram and its detection involves an understanding of algebraic variables. The students are aware that the teachers keep levels of mathematical work high and they appreciate it. When we interviewed students and asked them what it takes to be a good teacher, many of them mentioned the high demand placed upon them, for example:



She has a different way of doing things. I don't know, like she won't even really tell you how to do it. She'll be like, 'think of it this way'. There's a lot of times when she's just like – 'well think about it' – and then she'll walk off and that kills me. That really kills me. But it's cool. I mean it's like, it's alright, you know. I'll solve it myself. I'll get some help from somebody else. It's cool. (Ana and Latisha, Y3).

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The following students, in talking about the support teachers provide, also refer to their push for understanding:

Int: *What makes a good teacher?*

Patience. Because sometimes teachers they just zoom right through things. And other times they take the time to actually make sure you understand it, and make sure that you actually pay attention. Because there's some teachers out there who say: 'you understand this?' and you'll be like "yes" But you really don't mean yes you mean no. And they'll be like "OK" And they move on. And there's some teachers that be like – they know that you don't understand it. And they know that you're just saying yes so that you can move on. And so they actually take the time out to go over it again and make sure that you actually got it, that you actually understand this time. (John, Y2)

The students' appreciation of the teachers' demand was also demonstrated in our questionnaires. One of the questions started with the stem: ‘When I get stuck on a math problem, it is most helpful when my teacher ...’ This was followed by answers such as ‘tells me the answer’ ‘leads me through the problem step by step’ and ‘helps me without giving away the answer’. Students could respond to each on a four-point scale (SA, A, D, SD). Almost half of the Railside students (47%) *strongly* agreed with the response: “Helps me *without* giving away the answer,” compared with 27% of students in the ‘traditional’ classes at the other two schools ($n = 450$, $t = -4.257$; $df = 221.418$; $p < 0.001$).

2. Effort over ability

In addition to the actions in which teachers engage, challenging through difficult questions that maintain a high cognitive demand, the teachers also give frequent and strong messages to students about the nature of high achievement in mathematics, continually emphasizing that it is a product of hard work and not of innate ability. I have already described the multidimensionality of classrooms and the fact that teachers take every opportunity to value something students can do, but they also keep reassuring students that they can achieve anything if they put in the effort. This message is heard by students and they communicate it to us in interviews, with absolute sincerity, for example:

To be successful in math you really have to just like, put your mind to it and keep on trying – because math is all about trying. It's kind of a hard subject because it involves many things. (...) but as long as you keep on trying and don't give up then you know that you can do it. (Sara, Y1).



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In the year 3 questionnaires we offered the statement “Anyone can be really good at math if they try”, 84% of Railside students agreed with this, compared with 52% of students in the traditional classes ($n= 473$, $t = -8.272$; $df = 451$; $p<0.001$). But the students do not only come to believe that they can be successful, they develop an important practice that supports them in that – the act of persistence. It could be argued that persistence is one of the most important practices to learn in school – one that is strongly tied to success in school as well as in work and life. We have many indications in our data that the Railside students developed considerably more persistence than the other students. For example, as part of our assessment data we give students long, difficult problems to work on for 90 minutes in class, which we videotaped. The Railside students were more successful on these problems, partly because they would not give up on them and they continued to try to find methods and approaches even when they had exhausted many.

When we asked in questionnaires: ‘How long (in minutes) will you typically work on one math problem before giving up and deciding you can't do it?’ The Railside students gave responses that averaged 19.4 minutes, compared to the 9.9 minutes averaged by students in traditional classes ($n=438$, $t = -5.641$; $df = 142.110$; $p< 0.001$). This response is not unexpected given that the Railside students worked on longer problems in class but it also gives some indication of the persistence students are learning through the longer problems they experience.

In the following interview extract the student links this persistence to the question asking and justification highlighted earlier:

A: Because I know if someone does something and I don't get it I'll ask questions. I'm not just going to keep going and not know how to do something.

L: And then if somebody challenges what I do then I'll ask back and I'll try to solve it. And then I'll ask them: “Well how d'you do it?” (Ana and Latisha, Y3).

3. Clear expectations and learning practices

The final aspect of the teachers' practice that I will highlight also relates to the expectations they offer the students. In addition to stressing the importance of effort the teachers are very clear about the particular ways of working in which students need to engage. Cohen & Ball (2001) describe ways of working that are needed for learning as ‘learning practices’. The teachers are very clear about helpful learning practices and they frequently stop the students as they are working to point out valuable ways in which they are working (an example is given in the longer version of this paper). The teachers also spend time before projects begin setting out the valued ways of working, encouraging students to, for example, pick ‘tricky’ examples when writing a book (that is one of the projects they complete) as they will “show off” the mathematics that they know. The teachers communicate very clearly to students which learning practices will help them achieve. This was also true of the teachers in the school in England that I studied (Boaler, 1997, 2002) which also brought about more equitable outcomes.



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Relational equity

Many researchers are concerned to understand and promote equitable schooling practices. In studying equity most researchers look for reductions in achievement differences for students of different ethnic and cultural groups and genders, as well as equitable treatment in schools. Post (2004) refers to these two versions of equity as equal outcomes and equal access. The version of equity that I would like to consider in addition to these two important aspects of equity is closer to Elizabeth Anderson's conception (1999) that she refers to as 'democratic equity'. Anderson is a philosopher whose concern is not with test scores or other measures of educational achievement but with an individual's standing in society. 'Democratic equity is identified by an individual "standing as an equal over the course of an entire life" (Anderson p 319)' (from Post, 2004, p 11). This conception shares some aspects of the version of equity I would like to propose. In observing Railside classes over many years I have come to appreciate what I will term as *relational equity*. This conception of equity also goes further than students' test results in schools and considers the ways students are acting in school and the ways they learn to regard and relate to one another. Unlike Anderson's conception of equity, relational equity is about students' relationships with other students and with subjects and with the intersection of the two, as I shall outline below. In proposing this conception of equity I am asking the question – what would it mean to teach students to act in equitable ways in classrooms? This question was raised, for me, through observations of the Railside classrooms and the behavior of the teachers and students. Indeed it would be hard to spend years in the classrooms at Railside without noticing that the students were learning to treat each other in more respectful ways than is typically seen in schools and that ethnic cliques were less evident in the mathematics classrooms than they are in most schools. Further, such behavior did not just *happen* to take place in a mathematics classroom, it was fundamentally related to the students' conceptions of and work within mathematics. I propose the term *relational equity* because it is about students' relationships with each other developed through mathematical work as well as students' relationships with the subject of mathematics. Such relationships led to more equitable achievement results at the school as well, I contend, to more equitable ways of working that students would take into their lives.

Richard Schweder, a cultural anthropologist and cultural psychologist talks about the importance of considering different perspectives on issues in the working of a democratic society:

'It is often advantageous to have more than one discourse for interpreting a situation or solving a problem. Not only alternate solutions but multidimensional ones addressing "several orders of reality" or "orders of experience" may be more practical for solving complex human problems.'
(Schweder, 2003, p100)

The Railside students learn through their mathematical work that alternate and multi-dimensional solutions are important which leads them to value the contributions of the people offering such ideas. This is particularly important at Railside as the classrooms are multicultural and multilingual. It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues or read diverse forms of literature in English or Social studies classes. I propose that all subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject removed from responsibilities



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of cultural or social awareness, has an important contribution to make. For the equitable relationships that Railside students developed and that I have discussed more fully in Boaler (2004) are only made possible by a conception of mathematics that values the contribution of different insights, methods and perspectives in the collective solving of a particular problem with a particular solution. The relational version of mathematics that these students learn in which they come to value different contributions to a problem and different relationships between mathematical methods enables the respectful working relationships I have set out. It seems to me that it is important to consider whether students are learning to respect students from different ethnic and cultural groups and genders in our schools today, yet such concerns are not captured in notions of equity that are measured by test scores. Relational equity builds from the relations within subjects. As students learn to appreciate and make connections between different mathematical methods and ideas and to value the contributions of different perspectives on problems, given by students who think in quite different ways, they will also learn something extremely valuable about people, relations and ideas.

Conclusion

Railside is not a perfect place - the teachers would like to achieve more in terms of student achievement and the elimination of inequities, and they rarely feel satisfied with the achievements they have made to date, despite the vast amounts of time they spend planning and working. But research on urban schools and the experiences of mathematics students in particular tells us that the achievements at Railside are extremely unusual. In this paper I have attempted to convey the work of the teachers in bringing about the reduction in inequalities as well as general high achievement that they achieve. In doing so I hope also to have given a sense of the complexity of the relational and equitable system that they have in place. People who have heard about the achievements of Railside have asked for their curriculum so that they may use it, but whilst the curriculum plays a part in what is achieved at the school it is only one part of a complex, interconnected system. At the heart of this system is the work of the teachers, and the many different equitable practices in which they engage.

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