

Developing mathematical understanding

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Abstract

The notion of scaffolding is used to introduce research on differentiated learning trajectories that make use of activities termed “prompts”. The prompts are used to enable children to develop the necessary mathematical understanding and skills to keep up with the rest of the class. They focus on aspects of teaching that teachers identified as important aspects of classroom interactivity that contribute to the understanding: the use of physical representation of concepts: actions aimed at building conceptual links; and the use of language based activity.

Scaffolding learning

Vygotsky’s seminal work focused on how social activity mediates cognitive development. In socio-cultural theories that draw on this work, understanding is portrayed as developing through interpersonal, social activity that is shaped and constrained by the features and norms of the particular context in which it evolves. The specific classroom feature, aimed at developing understanding, that this paper focuses on is ‘scaffolding’.

The term ‘scaffolding’ grew out of Vygotsky’s research on how children learn. As a theoretical concept, it is used in two closely related ways in relation to school education. First, *cultural scaffolding* (Salomon and Perkins, 1998, p. 5) involves teachers’ and students’ use of social systems and tools. These include traditional ways of working as well as artefacts such as language, symbol systems, and books. In relation to mathematics education, researchers such as Cobb and his colleagues (e.g., Cobb and McClain, 2001) have described complementary mathematical and socio-mathematical activity of classrooms that tacitly embody such accumulated social wisdom.

A second, but inextricable, interpretation is *scaffolding of learning*. It is this second form of scaffolding that is the focus of this paper. Here, teachers plan pedagogical pathways and modelling that support children’s movement into new areas of knowledge and skill (Wood, Bruner and Ross, 1976; Bruner, 1983, 1986b). This form of scaffolding involves two interwoven processes - active guidance and modelling by the tutor, and active construction of knowledge by the learner. These two processes are not sequential, but in keeping with Vygotsky’s theory, are seen to be responsive and interactive features of learning contexts. “Guided participation necessarily involves subtle communication between people as to what new information is needed or appropriate and how it can be made compatible with current levels of skill and understanding” (Rogoff, 1991, p. 351).

Bruner (1996) wrote about this form of scaffolding as a logical structuring of ideas to be understood in an order that is likely to lead children to develop further and faster than they would on their own. Here, a mentor aims to build a bridge from

children's current understanding to reach new understanding through processes inherent in communication.

“That is, the teachers’ selective intervention provides a supportive tool for the learner, which extends his or her skills, thereby allowing the learner to successfully accomplish a task not otherwise possible. Put another way, the teacher structures an interaction by building on what he or she knows the learner can do. Scaffolding thus closes the gap between task requirements and the skill level of the learner.” (Greenfield, 1984, p. 118)

This idea is drawn from Vygotsky’s notion of the “zone of proximal development” (Vygotsky, 1978, p. 86), a term used in relation to development that can take place with the assistance of a mentor or more knowledgeable peer - the zone lying between the zone of actual development (what the child currently understands and can do) and the zone of potential development (where the child’s understanding and skills are headed in the longer term). Pedagogical activity within this zone is aimed at bridging the gap between what is known and what can be known with appropriate teaching.

Bruner (1986a) wrote that it is a “loan of consciousness” that gets a learner through this zone of proximal development (p.132), arguing that scaffolding by teachers facilitates “guided” participation. Rogoff (1991) pointed out that participation in the learning process should be pictured not as a function of simple internalisation, with modelling or information being transferred “across a barrier from a social partner to the inside of a child”, but through active social negotiation whereby learners transform their own understandings (p.362). In articulating Bruner’s notion of guided participation further, Rogoff, (1991) noted that, “Guided participation involves transfer of responsibility for handling more complex features of a problem as children develop skill and is, hence, a dynamic process of structuring and supporting development” (p.351).

Crossing the zone: Learning trajectories

Processes of structuring and supporting development in mathematics classrooms generally involve teachers making evidence-based assumptions about what is known about any topic and then planning a series of activities that should lead to specific learning outcomes. Cobb and McClain (1999), for example, identified the need for an “instructional sequence (that) takes the form of a conjectured learning trajectory that culminates with the mathematical ideas that constitute our overall instructional intent” (p.24). Wood (1991, p.109) described effective teaching as “leading by following”; noting that effective scaffolding of learning draws on the interests and understandings of students. Hiebert et al. (1997) called on the notion of learning as “residue”, involving the understandings children gained from prior teaching being used as a basis for further planning, with sequences of tasks being aimed at the development of further particular residues over time.

The emphasis in most writing about the scaffolding of learning has been on personal rather than social scaffolding. Lerman (1998) provided a challenge to this with his suggestion that the zone of proximal development may be thought of as a feature of the class as a whole - a characteristic of the mathematics classroom itself. Lerman (2001) also noted that a focus on scaffolding of learning does not attend to teacher learning. Simon (1995) brought interest in personal and the social scaffolding of the learning of both teachers and students together in his consideration of the idea



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of “learning trajectories”. He demonstrated how “The continually changing knowledge of the teacher (the teachers’ own learning trajectory) creates continual change in the ‘teachers’ hypothetical learning trajectory” (p. 141). Simon used this latter term to refer to teachers’ expectations of how students might develop specific mathematical knowledge.

“A hypothetical learning trajectory provides the teacher with a rationale for choosing a particular instructional design; thus, I [as a teacher] make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions ... as well as the spontaneous decisions that I make in response to students’ thinking. The hypothetical learning trajectory is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process - a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities.” (pp. 135-136)

I will return to the topic of learning trajectories after a short deviation to describe a research context.

What teachers claim they do to promote mathematical understanding

Over the past five years, I have carried out research into how teachers describe and develop students’ mathematical understanding. Case study methods have been used in this research agenda because case study allows researchers to deliver evidence of what happens at the interface of mathematics teaching theory and practice in real social contexts. Case study allows not only analysis of practice, of teachers’ and other stakeholders’ beliefs about that practice, and of the social context of that practice, but it also facilitates a study of interactions between these components. Tradition “tools” of mathematics education that can be captured for interpretation include mathematical and socio-cultural classroom norms as well as features of lessons such as specific learning trajectories.

In the study reported here, about 4 weeks were spent in each of 4 classrooms in one primary school. The aim of the research was to identify the social systems and tools that the teachers used in order to develop mathematical understanding. Each of these teachers was interviewed several times, and their mathematics lessons were videotaped. The resulting audiotaped and videotaped data were analysed to find examples of what the teachers believed and did in relation to the development of their pupils’ mathematical understanding. The full report of this research (Mousley, 2003) is descriptive, with close reference and electronic links to extensive multimedia appendices.

When the teachers were asked what they did to develop mathematical understanding, the data were categorised under emergent categories. The major categories were headed (a) the use of physical representation of concepts; (b) actions aimed at building conceptual links; and (c) the use of language based activity. Analysis of videotapes of the mathematics teaching of each of the subjects supported their claims about these three main foci, and it resulted in useful case materials of how each class of activity helped teachers to meet the relevant objectives. The videotapes also provide evidence of the problematic aspects of their use, as outlined below.



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The use of physical representations of concepts

The professional context of primary mathematics education in Australia has a strong emphasis on the use of what teachers call “concrete materials” or “teaching aids”. There is a huge body of research and literature on how to use these to aid the development of specific mathematical understandings (see, for instance, reviews by Perry and Howard, 1994, and Szendrei, 1996). Usually, the uncontested conclusion is that “Manipulatives help pupils develop and understand the concepts, procedures, and other aspects of mathematics” (Szendrei, 1996, p. 427). Activity with materials at lower levels of schooling is also well supported by policy documents, textbooks and duplicated worksheets, “how to” articles in teachers’ journals, pre-service and ongoing teacher education, and staffroom conversations. There is often very detailed and graphic advice available in (or accompanying) such resources and professional experiences.

The lessons observed provided ample evidence of effective use of manipulatives. However, each form of representation holds social meanings, so is both material and conceptual. For example, use of the physical MAB blocks, 10x10 grids, and metric tape measures all require an understanding of base 10 relationships. It has been recognised for a long time that manipulating physical objects without understanding their connections with mathematical concepts can be counter-productive, and that the shift from concrete material to abstract ideas is not an easy one. Even when tools are designed with this transition in mind, the logic of their structures cannot be taken for granted, for understanding relies on prior experience with the underpinning ideas. There were many times in the videotaped lessons when the children did not seem to map the physical representations against appropriate concepts - or perhaps did not map their manipulation against the mathematical operations being studied. At these times and for these children, the teaching aids seemed to provide a layer of complexity that they could not handle successfully.

Building conceptual links

All four teachers talked about the need to work at building students’ conceptual links, and the videotapes illustrated a variety of ways of doing this. One approach involved helping learners to make a connection between new information and existing understandings, and this involved frequent use of recall as well as careful structuring of activity sequences. A second way of building conceptual links was to articulate and demonstrate relationships between different mathematical ideas and representations, such as decimal fractions (Number) and the tape measures and metre sticks used in Measurement activities in the classrooms. A third focus that the teachers had was on emphasising links between school concepts and the mathematical aspects of other everyday contexts, and this was particularly noticeable in problem-solving activities.

Again, it was clear that the use of each of these approaches to building conceptual links needs careful consideration because some of the connections that seemed obvious to the teachers proved difficult for children to make. It was clear that some of the children did not see “obvious” relationships between operations, between concepts and everyday experience that had similarities. Others had unexpected difficulty relating new knowledge to what had been learned and used successfully in previous lessons.

The teachers’ planning of structured sequences proved quite problematic because students did not necessarily have knowledge that was assumed at the beginning of lessons or because they could not make the necessary conceptual leaps as lessons (or series of lessons) progressed. The work of Lave and Wenger (e.g. 1991) - as well as

other researchers who have explored ideas of situated cognition - proved very relevant to analysis of these results (see Mousley, 2003).



Language-based activity

The final category of strategies used by the teachers to develop students' mathematical understanding encompassed language-based features of the mathematics teaching and learning. Most commonly, these were (a) explanations given by teachers or requested of the pupils; (b) encouragement of children's verbalisation of solutions and mathematical discussions during co-operative working in small groups; (c) questioning; and (d) the use of open-ended verbal problems. It was proposed that such verbal activities not only facilitate the sharing of ideas and processes but help children to clarify their own ideas through speaking them. Further, different forms of verbal activity were seen by the teachers as providing avenues for communication about what is or is not understood.

Talking about current understandings, exploring new ideas aloud, and either stating or asking questions about things that are not understood are all processes supported by the research literature (e.g., Ball and Bass, 2000; Clarke and Atkinson, 1996). Again, however, the use of such strategies is not unproblematic. For example, research by Rogoff and Gauvain (1988) demonstrated that having an adult partner rather than a peer led to better collaborative processes and better independent performance in the future. They attributed this to adults structuring learning tasks and discussions in ways that made concepts, processes, and sequencing of ideas transparent. They also proposed that the presence of peers was, at times, a distraction and that division of labour was more difficult to organise with young children.

In summary, it seems clear throughout this research that teachers are aware of ways that they can facilitate the development of mathematical understanding, and that they put these to good use in their mathematics lessons. However, it was also evident that when to use - and how best to use - such teaching strategies is open to question.

Scaffolding and the development of mathematical understanding

The approaches above are commonly cited by teachers and widely recommended in professional literature for improving the learning of mathematics, but their implementation may actually disadvantage particular groups of students. For instance, heavy emphasis on the use of manipulatives may hold back children who already have strong abstract understanding. (Indeed, the videotapes show instances of children quickly solving a problem in their heads, recording the answer and then spending time building models with blocks as expected.) Similarly, emphasis on verbalisation may make mathematics lessons harder for students from non-English speaking backgrounds (as illustrated, for example, by Chan, 2004). Researchers (e.g. Cooper and Dunne, 1998; Mousley, 1992; Sullivan, Zevenbergen and Mousley, 2002) have also demonstrated how contextualising mathematics tasks may create particular difficulties for low socio-economic status students. Likewise, open-ended problems add challenges that some students find more difficult than others (Lubienski, 2000).

Thus in using what are commonly thought to be effective approaches to teaching mathematics, perhaps we are creating unintended barriers for some students.

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Overcoming barriers in learning trajectories

I have been working with Peter Sullivan and Robyn Zevenbergen to explore ways in which typical features of today's mathematics classrooms may provide barriers in the learning trajectories of some students (see, for instance, Sullivan, Mousley, Zevenbergen and Turner Harrison, 2003; Zevenbergen, Mousley and Sullivan, 2004). Using action research, we are working with teachers in schools where there is a mix of socio-economic backgrounds, with most schools involved having significant numbers of indigenous and immigrant students to explore ways that issues arising from a range of differentiated needs can be addressed.

Part of this research has involved consideration of what we have called "alternative learning trajectories". The aim here is for teachers to develop hypothetical learning trajectories that they think are appropriate for the whole class. End-of-lesson learning goals are common for all students, and it is expected that all will be able to take part in summative discussions as well as to leave the mathematics classrooms with a sense of success. However, along the teachers' hypothetical learning trajectories, alternative pathways are carved out at points where students may meet barriers. At these points, "enabling prompts" are used. These are social tools (activities or interactions) that provide conceptual stepping-stones back to the main trajectory of activity that leads to the learning goal. Initially we designed some model lessons using this approach, and now teachers in the project who have grasped the idea are using it in their everyday teaching.

As an example of enabling prompts, one of the model Grade 5 lessons, *Area as squares*, involves the use squared paper so that the children can count the units in order to assist them in calculating areas of rectangles and triangles. The aim of the lesson is for them to deduce and to develop good understanding of the area formulae for these shapes. The lesson first has the children draw, on squared paper, letters of the alphabet using exactly 10 whole squares. *As a prompt for students experiencing difficulty, some squared papers with one letter already drawn are available.* The students are then asked to draw, on another piece of squared paper, other letters of the alphabet using a total of 10 squares, using some half squares to make the letters easier to read. *As a prompt for students experiencing difficulty, squared paper with a letter O drawn on it is available.* Students then complete a worksheet that involves working out the area of some given colored rectangles. *Students experiencing difficulty are given a worksheet with the lines in the first rectangle shown, dividing it into unit squares that can be counted.* The second task on this same worksheet is to work out the area of some (right-angled) triangles. The most demanding aspect of this task uses triangles where the hypotenuse cuts through a number of the squares so they are not easily counted. The final challenge for the students is to draw many different triangles, each with an area of 12 square units. The model lesson then asked that some students come to the front of the room and explain what they have done.

It is important to note from the above lesson description that in each case where a prompt is available, (a) the same teaching aid is used, but one level of conceptual processing is removed; (b) after a quick activity using the prompt the students rejoin the class' learning trajectory immediately; (c) students choose to collect the prompt sheets as needed, without a public display or discussion of their not understanding the mainstream task. Also, the learning trajectory is carefully sequenced from a simple exercise to a very challenging one that requires mathematical imagination and generalisation. It is expected that all students will complete the range of tasks that should develop their understanding of area as squares and of the processes for calculating the area of squares and triangles. We have found that most Grade 5 children can make this



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transition successfully, even though their teachers initially think that the tasks will be too hard for some of them.

In the lesson above, the use of enabling prompts is a scaffolding technique applied to the use of a physical aid (squared paper). However, enabling prompts are also proving effective in helping children to make connections between mathematical concepts. Further, they are proving effective when aspects of language use provide barriers to successful completion of lesson activities. Most importantly, we have found that the use of differentiated learning trajectories generally allows all children to understand and to make useful contributions to discussion, sharing, and display of their work at the end of the lesson. Further, the meeting of classroom based learning objectives also provides a strong foundation for the next step in the scaffolded learning process.

Conclusion

Vygotsky claimed that the development of the mind results from goal oriented and socially determined interaction between human beings and learning environments. The challenge for researchers and teachers of mathematics education is to find out what sorts of interactions and experiences best lead to the development of mathematical understanding.

Scaffolding of learning does not merely involve constructing a sequence of activities for a class - or even the unattainable goal of planning suitable learning trajectories for 30 little individuals. It suggests teachers' and students' use of tools and activities that enable students to be drawn forward, with appropriate mentoring, into new area of experiential and cognitive growth. We are finding that it is reasonable to expect that teachers will develop hypothetical learning trajectories for their students as a whole class, so long as they plan activities that will help students meet barriers along the way. Such barriers are likely to be associated with the very processes that teachers use to develop mathematical understanding: the increasing use of physical representation of concepts; actions aimed at building children's conceptual links; and a growing use of language based activity in mathematics classrooms.

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