Come away with me: Statistics learning through collaborative work[1]

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Introduction

In a more technological and literate society statistics plays a relevant role (Batanero & Godino, 2001), namely to allow people to become critical and active citizens. Statistics is part of our daily life. Most *media* refer to statistical knowledge showing graphs, tables or means in order to sound as being scientifically supported and to be able to manipulate people's opinions about what is going on in the world. Choosing the information one reads, analyzing it, processing data and deciding different ways of presenting them are some of the competencies that we need to mobilize and develop (Gal & Garfield, 1997). Otherwise one experiences exclusion at a more or less visible level, but still affecting one's power to participate in social practices, like voting, deciding what to buy, how to pay taxes, or even about recycling – or not – one's waste. In this sense, "Statistics is seen like a tool allowing for understanding and interpreting the world around us, thus contributing to the education of autonomous, critical, and participant individuals (...)" (Brocardo & Mendes, 2001).

But, as Lajoie, Jacobs, and Lavigne (1993) stress, not only adults need to be literate and critical about statistical knowledge. According to these authors, children (and teenagers, we would add) also need to become more active and critical citizens. Thus, they need to have access to statistical knowledge that develops complex functions (Vygotsky, 1932/1978, 1934/1962) and not only to meaningless procedures they mechanically learn to use (Mokros & Russel, 1995), even if some authors state that pupils usually begin by using procedural competencies before achieving relational ones, mainly due to their previous instruction in statistics (Cobb & McClain, 2004).

School practices play an essential role in the access pupils will have, or not, to these forms of literacy so deeply needed in a complex, changing and multicultural society. In the 21st century most countries and schools are becoming increasingly multicultural. Thus, diversity is a reality teachers must accept and respect (D'Ambrósio, 2002; Favilli, Oliveras, & César, 2003). In most classes there are pupils from different cultures and home backgrounds, with diverse mathematical traditions, ways of reasoning, and ways of solving mathematical tasks. Some cultures are quite far from the mainstream one, and also from the school culture (César & Favilli, 2005). Thus, being able to use diversity as richness instead of avoiding it is one of the main challenges teachers face nowadays (César, 2003).

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Portuguese policy documents clearly suggest that mathematical activities should provide a diversity of learning experiences in order to facilitate knowledge appropriation as well as the mobilization and development of competencies (Abrantes, Serrazina, & Oliveira, 1999). As concerns statistics knowledge the recommendations include using project work, group work, and other ways of confronting pupils with daily life use of statistical information available in their social and cultural settings. But although these ideals are already expressed in policy documents teachers classroom practices are still quite far from them, despite several research reports providing empirical evidence that there are also some very innovative practices co-existing with the more traditional ones (Ponte, Matos, & Abrantes, 1998). On the other hand, mathematics is still a very selective subject concerning vocational and professional choices, and it also presents a high rate of underachievement.

In the Portuguese school system, statistics is part of the mathematics subject from the elementary school level until the 12th grade (the last one before university level). Many teachers still do not value statistics contents very highly (Ponte & Fonseca, 2001), and/or feel uncomfortable exploring them because, according to their own words, during their pre-service education, statistics received little attention (Oliveira, 2004). Although there have been several curricular changes during the last decades, Ponte and Fonseca (2001) state

> Curricular change does not end with the elaboration and legislation of official documents. It includes, as we know, the production of very diverse materials, teacher education, the study of pupils' difficulties, and of the conditions needed in order to achieve these new proposals. In this domain, in Portugal there is a lot to be done considering the little attention paid to this topic [statistics]. The needed changes include, above all, a change in perspective, no longer viewing statistics as a "poor" and uninteresting chapter in Mathematics, but rather regarding it as an essential element in citizens' basic education. (p. 112)

As we wanted to promote a change in statistics learning and instruction, one of our main concerns was to propose challenging, meaningful, and contextualized tasks. But our ultimate aim was to contribute to a more inclusive schooling (Ainscow, 1991, 1999), able to respond to all and each pupil's needs and offering them real learning opportunities. Thus, we decided to use problems or project work (Abrantes, 1994), and also to establish a new didactic contract from the beginning of the school year, based on collaborative work (César, 2003). This new didactic contract was conceived as a mediational tool (Vygosky, 1932/1978) for knowledge appropriation as well as for the mobilization and development of competencies.

This research, using an action-research approach, took place in several classes all over the country as the *Interaction and Knowledge* project began in 1994/95 and will last until 2005/06. We analyze a peer interaction (4 pupils per group) from a 9th grade class (for pupils who never fail this grade corresponds to the last one of compulsory education) related to probabilities, and also some examples of project work related to statistics, from four different 10th grade classes. Both cases illuminate empirical evidence that stress the facilitating role of the nature of these tasks and collaborative work in pupils' development and performances. Pupils' engagement in the new didactic contract also plays an important role in their knowledge appropriation, as illustrated in interaction excerpts and in pupils' own voices (interviews and questionnaires).

Theoretical background

Nowadays most international and Portuguese policy documents indicate that school should provide the means to develop pupils' competencies, namely those related to communication and to participant citizenship (Abrantes et al., 1999; Ministério da Educação, 2003; Ponte & Fonseca, 2001). But in order to achieve these aims teachers should pay special attention to the nature of the tasks they propose, namely in statistics. Zawojewski (1991) states that there are different ways of collecting data for statistical treatment and analysis, namely surveys, experiences and simulations. Pupils can be engaged in data collection by working on questions that are meaningful to them. This author stresses the long-term character of these types of activities, but also that learning becomes relevant and meaningful, facilitating a global approach to problems, and even to the learning process. Cobb, McClain and Gravemeijer (2003) developed longitudinal studies related to statistics learning and stress the need for long term studies in order to understand pupils' statistics learning process. Batanero and Godino (2001) go even further, relating the statistics learning process to the research process itself. In order to achieve this goal, they suggest that "in statistics learning we can use small investigations that contextualize learning and facilitate pupils' understanding about statistics' role in the broader process of research" (p. 15). In both cases pupils' meaning attribution plays a fundamental role, stressing the social and cultural elements as part of the learning process.

Assuming a sociocultural perspective, Cobb and McClain (2004) point out that considering reasoning as situated allows for viewing pupils' statistical reasoning not merely as documenting an inherent psychological stage, but also as documenting the quality of their prior instruction. Carvalho (2001) also states that prior instruction shapes pupils' solving strategies when they are solving statistical tasks. This illuminates how careful researchers must be in their interpretations, as pupils' performances are always historically and culturally situated. But in addition, in another study, Cobb and Hodge (2002) stress that

> students' development of identities is an integral aspect of their engagement in the activities of a community such as that constituted by the teacher and students in a classroom (Wenger, 1998). Further, students' emerging identities as doers in mathematics are viewed as forms of individuality that are defined with respect to the types of competences that membership in the classroom entails. (p. 1)

César (2003) assumes that the way these identities are constructed is linked to the types of practice implemented in classes but they are also shaped by the didactic contract (Brousseau, 1988; Schubauer-Leoni, 1986), and by the coherence between these two elements. Identities are conceived as dialogical (Hermans, 2001). Each pupil has several dialogic identities that sometimes are even conflicting, namely with respect to mathematics, or school.

Knowledge appropriation as well as the mobilization and development of competencies are conceived as a complex process, shaped by social, emotional and cognitive elements, interacting in a dialectic way. Learning is viewed as a participatory, interactional process that includes the negotiation of cultural practices (Vygotsky, 1932/1978, 1934/1962; Wertsch, 1985, 1991). For instance, Cobb (1999) indicates that while doing statistics during classroom activities one could notice changes in the way pupils participated in the discussions, as the pupils progressively assumed more responsibilities in the process of generating data. But learning is also consequential as it has consequences both for the learner and the settings and situations in which he/she acts, reacts and/or responds. In this perspective, literacy is conceived as knowledge in action.

Collaborative work is also suggested in many policy documents, namely related to statistical contents (Abrantes et al., 1999). Piaget and Vygotsky (Tryphon & Vonèche, 1996) underlined the role of communication in knowledge appropriation and in pupils' performances. Vygotsky (1981) stresses the need to (re)construct meanings since knowledge is socially constructed and pre-existing to individuals, which means that it needs to be deconstructed and then reconstructed in order to become meaningful to a given person. Social interactions, namely peer ones, play a major role in this process, although what is meant by collaborative work changes across different research reports and papers (van der Linden, Erkens, Schmidt, & Renshaw, 2000). When we consider that we implemented collaborative work we have three different levels in mind: among peers, during class activities in which pupils are stimulated to interact in dyads or in small groups in order to co-construct solving strategies and answers; among teachers and researchers (some of them take both positions), when they are conceiving tasks, reflecting upon their practices or analyzing data; and, at a lower level of achievement, among research groups (César, 2003; César et al., 2001). Dyad or small group peer interactions while pupils are solving problems (César, 1998), or investigative tasks (Branco, Matos, Ventura, & Santos, 2004), as well as when they are participating in the whole class discussion (César, Mendes, & Carmo, 2001), contribute to knowledge appropriation, and also to the development of competencies, such as making conjectures, arguing, respecting one another's viewpoint, sense of responsibility and autonomy, and a critical sense, among others.

Vygotsky (1985) also states that having the possibility of working in the zone of proximal development is one of the advantages of social interactions. Moll (1990) adds that teachers should propose tasks that allow pupils to work within their ZPD. Rogoff, and Wertsch (1984) also discuss the advantages of learning within the ZPD and the conditions that facilitate this learning process. César (1998, 2003) goes further and considers that peer interactions could be seen as a facilitator when used within an innovative and coherent didactic contract as they facilitate knowledge appropriation and competencies development not only for the less competent peer, but also for the more competent peer – an aspect which constitutes one of the most fascinating pedagogical features of interactive practices. Peer interactions are seen as a way of sharing, and co-constructing mathematical knowledge, as well as statistics knowledge (Carvalho, & César, 2000a, 2000c, 2001; César, 2003; Schubauer-Leoni & Perret-Clermont, 1997). This also includes tasks whose nature facilitates peer interactions, as well as the need to construct an intersubjectivity (Wertsch, 1991), seen as an essential step towards communication.

Developing project work in statistics, namely when associated to collaborative work, is another way of promoting legitimate participation (Lave & Wenger, 1991), contributing to pupils' engagement in academic activities that are meaningful to them (Abrantes, 1994; Martins, Santos, Ferreira, & César, 2003). Long-term projects (around six months) allow pupils to experience every step of this process: from choosing the theme and elaborating questionnaires to presenting the results to their class, and then to their educational community. This engagement even in the decision making process makes them feel more responsible for the whole work, developing competencies that would not be needed otherwise.

Thorme and Root (2002) implemented a different kind of project that they call community-based research projects. They state that "the nature and structure of these community-based research projects facilitate realizing our most important learning objective of the course: pupils' gaining an appreciation for the statistical process and the nature of statistical inquiry" (p. 1). They also add that these projects help pupils' engagement and that in future studies they would like to explore the role of reflection and emotional engagement in pupils' construction of meaning. These authors wish to link constructivist principles with emotional elements in order to understand how they interact in statistics learning, pointing to another important advance: to consider also emotion as shaping the learning process, thus no longer studying only cognitive or social elements. A new challenge teachers and researchers must face in the years to come.

Method

Since 1994/95 we implemented the *Interaction and Knowledge* project in several mathematics classes (5th to 12th grade). Its main goal is to study and implement social interactions, namely peer ones in the classroom context. This project has a long history: the first studies (César, 1994; César, Perret-Clermont & Benavente, 2000) were *quasi experimental* ones and aimed at comparing performances, identifying diverse solving strategies and studying the academic progress of pupils working in different types of dyads or individually. It also aimed at better understanding the effectiveness of the different types of dyads, of "usual" and "unusual" tasks and of the working instructions that were given to pupils.

But these first studies already had an ultimate goal: to implement an action-research project in which researchers would work directly with teachers and which would promote pupils' positive attitudes towards mathematics, their socio-cognitive development and their school achievement. These first studies were related to equations (7th grade) but we knew that they should be expanded to other contents and school grades. We also realized that implementing an action-research project had to go hand in hand with continuing the *quasi experimental* studies in order to be able to construct a more sustained theoretical framework as well as to assume several moves between theory and practice that Perret-Clermont (1980, 1992) had already pointed out as essential for knowledge construction. But changing from the *quasi experimental* studies into an action-research project meant challenges, far from easy to overcome. For instance, we decided to implement a long-term follow up (10 years, for some selected classes and pupils) in order to be able to study the evolution of these pupils during the next years. And some of the stronger empirical evidence also come from this part of the research project, in which the selected classes were divided in three conditions: (a) classes whose pupils answer a questionnaire and privileged informers are interviewed every year; (b) a similar procedure, but only 5 and 10 years after they left the project; (c) a similar procedure only 10 years after they left the project. This means that we also wanted to see if the results we get with those from condition (a) are not too contaminated by seeing them so often.

The next content explored in the *quasi experimental* studies was statistics (7th grade) and once again peer work was a facilitator for knowledge appropriation (Carvalho, 2001; Carvalho, & César, 2000a, 2000b, 2000c, 2001). These second *quasi experimental* studies learned a lot from the action-research level of the project. For instance, results were even stronger because after the dyad work a general discussion was also implemented.

Until 2004, the *Interaction and Knowledge* project is divided into two levels: (1) *quasi experimental* studies that constitute a micro-analysis level, and that we may consider as semi-laboratorial, although contextualized, studies; (2) an action-research level, in which 42 teachers and psychologists study and implement collaborative work, namely peer interactions, in their classes. In the beginning of the project those classes were always mathematics classes. But as time went by they also included science, philosophy, and history. This also means that the research group includes teachers with quite diverse pre-service education, from different domains of knowledge, and also with differentiated levels of academic qualifications, including post graduation. Thus, one of the goals of this project is also to promote collaborative work among teachers, and among teachers and the researchers. It is a clear concern to promote teachers' personal and professional development, which also explains how the group works, and why collective reflective moments, seminars, material elaboration, in-service education sessions for other teachers, or the discussion of ways to disseminate results are regarded as so important by all the research group members.

Participants include the teachers and pupils whose classes are part of the project, but also external evaluators, as well as other members of the educational community of each school. There is a very large *empirical corpus* which means that we had to select a very small amount of that to present. The decision was to illustrate two different types of work: (1) a group interaction related to probability contents, audio taped in a 9th grade class (elementary school, as compulsory education lasts until pupils are 15 years old; expected ages: 14/15 years old) in a school in a suburb of Lisbon. This is considered a paradigmatic case, among many similar others, that illuminates the contributions of collaborative work, associated to problem solving both in statistics knowledge appropriation, and in the inclusion of pupils categorized as SEN (Special Educational Needs) in regular schools; (2) some examples of seven project works developed in two different schools, one also in a suburb of Lisbon and another in a town in the countryside (Caldas da Rainha). Project work was developed with 10th grade pupils (secondary school, expected ages: 15/16 years old).

The data were collected through participant observation (different observers, including external evaluators); video and audio taping; digital photos; questionnaires, interviews, pupils' work, academic documents, and teachers' and evaluators' reports. Special attention was given to certain cases in each class and we will discuss some of those related to statistics learning.

The procedures always included working collaboratively during at least a whole school year. In order to achieve this, the first week of classes was changed, according to the new didactic contract. The main aim of this week is to disclose pupils' interests, academic paths, life projects, and their mathematical competencies (and not merely their mathematical knowledge related to the contents they learned in previous school grades). Usually pupils have two 90-minute classes a week. In the first 90 minutes they answer a task inspired by projective techniques whose main aim is to study and understand pupils' social representations about mathematics; a small questionnaire about themselves (academic and leisure interests, life project, expectations concerning mathematics classes), and a task that evaluates their mathematical competencies. In the next 90-minute class the main aim is to promote positive academic self-esteem, as many pupils strongly reject mathematics, and they are so convinced they are unable to learn it, that they do not even answer or do anything concerning mathematical tasks. So, every pupil goes to the blackboard and explains to his/her peers, and teacher, one of the previous class competencies tasks (s)he solved successfully. For many pupils this is the very first time that they go to the board in a mathematics class, and that they do something right. Thus, later on they often refer to this first week as a very important experience.

After this week, they begin working in dyads, and they will work collaboratively in almost all activities. After that, there is a general whole class discussion. This means that both when this group interaction was audio taped (2nd term, February), and when project work was implemented (2nd and 3rd terms, from January until June, usually every 15 days), pupils were already used to working collaboratively. In previous papers we discussed peer interactions (dyad work) that were audio taped during the third class in which pupils were interacting and they allow for understanding how this process changes as time goes by, and when pupils really begin to appropriate the new didactic contract (César, 1998, 2003).

Project work was developed during the same months in which pupils were also studying functions. They had a class, approximately every 15 days, in which they did project work. In all the other mathematics classes that took place during those six months they explored functions.

From the beginning of the project work they knew how many classes there would be for statistics, as this is decided by all the mathematics teachers of that school, according to the suggestions of the curriculum. They varied from twenty 50-minute classes (before the last reform that changed classes into 90-minute ones) to thirteen 90-minute classes, considering the seven schools included in project work. Pupils began by choosing the theme they wanted to address, and the sample they would use (a representative one). Then, they elaborated the questionnaire. All these steps were discussed in groups, and then by the whole class. After this, pupils applied the questionnaires according to the sample they had selected. This was the only part of the work that was done outside the class. After collecting the data through the questionnaires they had several classes to treat and analyse them. But they also had to decide how to do that. In the end, they prepared posters to present their work to their class, and also to the educational community.

All documents and photos used in this paper were selected and authorised to be used both by teachers/researchers and pupils. Thus, this is another feature that makes this work truly collaborative, even as regards the dissemination of what was done in this research project.

Results

This four-pupil group was working together for the first time in this class. Before, they had worked as two separate dyads and this is still visible in the way they interact: (Dyad 1) T. is more concerned with J. and she keeps asking him questions to be sure he understands what is going on (Turns 11, 13, and 28). She is also the one who is immediately able both to calm down N. (Turn 33) and to understand what J.'s reasoning is when he gives the only wrong answer he suggests (Turn 31). So, she is able to redirect his reasoning by suggesting him to go step by step, and also by asking him questions that guide the way he should reason (Turns 35, and 37). This reveals a well developed social competence to follow others' reasoning, even when different from your own, and also to know what you should do in order to help them progress. It shows that T. is able to understand how she must deal with each member of her group in order to potentialize each one's contribution; (Dyad 2) N. is used to working with A., who used to be a pupil who rejected mathematics, and who never succeeded in this subject until the 8th grade, when she began working collaboratively. So, N. easily had the leading role in his dyad. However, he accepts to share this role with T. (Turns 21, 24, and 45), and A. also collaborates with T. in promoting J.'s performance (Turn 40, and 42). Thus, what we can see is that passing from dyad work to group work also changed the interactive patterns that we used to observe before, although some features, related to the didactic contract, remained the same, like the ones that make all members feel responsible for the solving strategy they choose, and the solution they find.

If we analyse this interactive episode, we notice that, from the beginning, J. is trying to be a legitimate participant in the group and not a peripheral one (Lave & Wenger, 1991).

Problem: At the school bar drinks are all sold at the same price. Ricardo likes drinking either orange juice or coke. Pedro always picks orange juice, coke or pineapple juice. João drinks pineapple, orange or passion fruit juice. What is the probability that the three friends go separately to the bar and without planning anything beforehand, pick the same drink?

1. J - Do you know how to solve this?
2. T - I'd do a scheme!
3. J - A scheme? How?
4. A - I also think it might be… if we put the names of each of them…
5. N - Of course! We do one of those schemes with circles!

[At this point there are already three pupils trying to write on the sheet. T. was the fastest to take the sheet and writes as she talks]

6. T - Ricardo, Pedro, João...
7. A - Now...
8. N -Now you have to put down what each of them drinks… with those arrows…
9. T - I think I'd do it this way: on R which is for Ricardo I'd say that he could drink orange juice or coke…
10. N - Right… and you can also just write L [Laranjada, in Portuguese means orange juice] *or C, to be faster!*
11. T - Are you getting this, J.?
12. J - I Think so…
13. T- So do you know how to go on? [Silence] *Can you tell us what to write now?*
14. J - It's like we did with Ricardo… I mean… from Pedro's circle there are 3 lines, one for each drink: orange juice, coke or pineapple, and we can also just put the first letter, so as to have a neater scheme!
15. N - Hey T., what a great teacher you are!?! [Laughter]
16. A – I also know how to continue. Now we do the same for João. [Grabs the pencil and continues the scheme on the sheet]

One of the most interesting features of the beginning of this peer interaction is that J. feels confident enough to assume that he does not know how to solve that task (Turn 1). His question leads to a co-construction of a solving strategy, mainly between T. and N., but also with A.'s contribution (Turns 2 to 10). They all decide to do a scheme and they all suggest ways of operationalizing this idea. This means that although they were studying probabilities for the first time in the 9th grade, according to the national Portuguese curriculum, they felt confident to look for solutions, and they were able to make connections between this problem and other learning situations they had experienced before in mathematics classes.

But after a first part of great enthusiasm, T. realizes that J. is not participating. So, she decides to confirm he is following their solving strategy (Turns 11, and 13). Her attitude makes J. enter their talk and go on with this co-construction (Turns 12, and

14). A. also feels she should make a contribution. So, she is the one who draws the graphic representation of the scheme (Turn 16).

The next interesting feature is that J. is not satisfied with merely answering this part of the task. He uses mechanisms to lower the concern the others have about him and his performance (*So far so good, I've understood it* – Turn 17), but the tone and the smile in his face also show that this is an indirect way of giving himself positive feed-back. It also means: I was able to go one step further; now, let's proceed. And he asks the next question: Shouldn't they quantify the probability (last part of Turn 17).

17. J – So far so good. I've understood. But don't we have to give a number to show the probability?
18. N – Of course we do! But now it's easy. Look at this scheme we've done and see which is the only chance of them all drinking the same drink.
19. J – Just the orange juice... only two drink coke and pineapple too.
20. T - Right... and only João drinks passion-fruit juice; so that's no good either.
21. N – Then let's put a red circle on L, to show this is the only hypothesis that fits what we want... and we have to put captions on this so the teacher understands what we were thinking! [They draw a red circle around the Ls]

22. T – These are the favorable cases, right?
23. J - Ah! I remember that! Those are the ones you put on the top part of the fraction, right?
24. N - Yeah... But I don't know if you can put it that way exactly... we have to look in the book and see if that's the right definition...
25. T – But we also need...
26. A - ... all the possible cases...
27. N – But with the scheme that's easy to see them!
28. T – Yeah, but let's see if J. knows which ones they are.
29. J – All the possible ones?
30. N - Yes.
31. J – If we count the bottom ones, they're eight.
32. N - Eight?
33. T - Yeah... he talked about counting the drinks each one could drink!
34. N - But...
35. T – Don't think everything out all at once. Think only about Ricardo. How many favourables are there?
36. J - One. Drinking orange juice, which is what we marked with the red ball.
37. T – And how many are possible for Ricardo?
38. J - Two: orange juice or coke.
39. N - Great!
40. A – So we write...
41. J - 1/2.
42. A - Right!
43. J - Ah! I see! For Pedro it's 1/3 and for João it's 1/3.
44. T - Yeah.

45. N – And to know the probability of all of them drinking orange juice, what do we have to do with those numbers?
46. A – Multiply them!
47. T – Well done, A.! This one was for you! Just to see if you were paying attention!
48. A – Hey, I'm sorry. I didn't remember I wasn't supposed to answer first.
49. J – Forget it, I realized that. If I go to the blackboard, I can explain everything well: I do the scheme and explain what each thing represents. Then, under each name I write 1/2, 1/3 and 1/3 and at the end I show the calculation for finding the probability: 1/2 x 1/3 x 1/3 = 1/18. So, there's one favourable case, which is all of them drinking orange juice, but there are 18 possible cases.
50. N – Great! You're becoming real clever!

After J.'s question, N. tries to follow T.'s way of interacting with J. So, they both share the leadership of the group, and until J.'s mistake, they begin another co-construction of the solving strategy (Turns 18 to 30). During this part of their talk we see that the pupils already know what are the favourable cases, and the possible cases, although these notions become clearer in their talk when T. tries to make J. realize his mistake in order to facilitate his understanding of the right answer.

When T. is orchestrating J.'s answers, and leading the interactive process (Turns 33 to 41) we also notice that there are more evaluative comments that are used as positive feedback to J. (Turns 39, 42, and 44). This is also an interesting modeling example as, according to our field notes (participant observation by different observers), their teacher very seldom produces evaluative comments. And when she does it is to give positive feedback to pupils who have more difficulties solving mathematical tasks. With respect to pupils in general, she usually shows enthusiasm, she smiles. We can see that she feels proud of them and of their work that she is really confident about their competencies and performances, and she listens to them very attentively. But she clearly avoids being evaluative because, as she explained to us in the interviews, she believes that "*comments that are too evaluative make pupils depend on teachers' comments, and I want to promote autonomy, and sense of responsibility*".

We can see that three of these pupils already knew how to calculate the probability (Turns 45 to 47), but also that J. quickly understood what should be done (Turn 49). In fact, in the next classes as well as in the next individual written test he was able to use probability concepts, and to explain them during the group work, the general discussions, and even in written form by himself, during the individual test.

The easy-going atmosphere of this group is illuminated in many parts of this episode, especially when J. immediately calms down A. when she answers in his place (Turn 49). They all know that in this didactic contract each member of a group can be asked to go to the blackboard in the general discussion. J. also knows that he has more difficulties in learning than his group colleagues. So, he immediately decided to do a simulation of what he would say if he was the one chosen by his teacher to present his group's work. This way of acting shows three different essential aspects: (1) he feels confident enough to know he can explain everything right, which illuminates how much he implemented his academic positive self-esteem and mathematical performance since the beginning of the 8th grade (before, he used to sit quietly and do nothing at all); (2) he was really able to follow the solving strategy the group had developed; and, (3) more than that, he really wants to be an effective participant in that learning community (Lave & Wenger, 1991), he does not merely want a side place. And, of course, he also wants his peers to be sure that he is able to perform this task well, to make them feel as confident about his performance as he already is.

This analysis also illuminates several aspects that contribute to create more inclusive settings in statistics classes when we promote collaborative work. These pupils were in the 9th grade and were working collaboratively since the 8th grade. When we were asked to include this class in the project this request arose because J. was categorized as presenting SEN (Special Educational Needs) and he was deeply rejected both by his peers and by some of his teachers. However, when we read this episode we no longer notice any rejection. It illuminates a great ability to deal with diversity: different mathematical and statistical content knowledge, working rhythms, argumentation competencies and even academic achievement. It seems clear that working in dyads or in small groups within a coherent didactic contract facilitates mutual respect, solidarity and the acceptance of diversity. Despite this diversity the pupils were able to develop an intersubjectivity (Wertsch, 1991) and to co-construct a solving strategy that they could all understand and explain to their colleagues, if they were asked to do so by their teacher during the general discussion that followed the group work.

When we analyze the interviews during the follow-up, after leaving this project and moving to another school, we notice that there is empirical evidence that this project left some remains. Although interpretations must be careful because the pupils were talking to us, and we were associated with the project, some of their most interesting comments referred to practices that were implemented among them when they did not know that the follow-up would take place, like the pupils who decided to go on working in dyads in the 10th grade, or their net of relations which included former colleagues. And J.'s words are quite clear. He really misses his 9th grade class, although he is succeeding in his studies:

> I really miss my 9th grade class. I don't think I'll ever have mates like those again. At the school I'm at now each one fends for himself/herself ... and I don't like it. It's as if everything's gone back. (...) The marks haven't even been bad. What I miss is someone who really likes me, like the maths teacher and my mates from my old class. (J., interview at the end of the 10th grade)

T. also states that she prefers working collaboratively, and that she is no longer very pleased with too much competition, although her marks are still very high during the 10th grade. And she also tells us, on her own initiative, that she still keeps seeing J.:

> "The marks have been great, but our class is so competitive ... I've got no patience for that type of thing anymore ... fortunately there's three other mates from last year. We formed two dyads straight away and placed ourselves strategically ... one behind the other ... Luckily the teachers didn't notice ... or maybe they just don't care about that ... and they let us be. (...) Can you believe I've been in touch with J. over the phone to know how he's doing and we've even had lunch together with a few other mates?! That was one hell of a class! I'll never have such a good one again." (T., interview at the end of the 10th grade)

These results illuminate that teachers' practices, the nature of the tasks, the existing didactic contract, among many other elements, can play an essential role in pupils' knowledge appropriation and the development of competencies, not only in the cognitive domain, but also in the social and emotional ones.

The next example is related to statistics learning during the 10th grade, and to the potentialities of developing project work associated with collaborative work. This was also a way of considering statistics an important topic that would neither be avoided, nor merely treated at the end of the school year, when classes are near the end. Usually 10th grade teachers value geometry and functions as the main contents to explore, and statistics occupies a very poor place in their concerns. Implementing this project work made statistics become much more visible to pupils, but also to the learning community, as they explained their work during the schools' exhibitions.

The constitution of the groups and dyads was the teachers' responsibility. They preferred heterogeneous dyads, or small groups, whose members' competencies and knowledge were complementary, facilitating their working within their ZPD (Vygotsky, 1932/1978; 1934/1962). Pupils accepted their choices easily because they felt so astonished and impressed by the first week of classes that they really wanted to see what would happen with this new method.

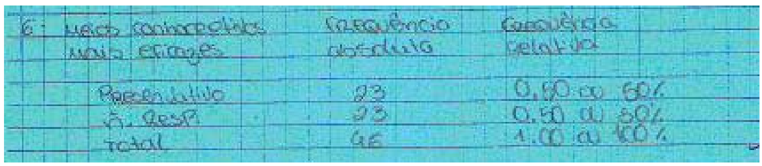
The choice of themes for the project works took some time and generated a long discussion, as each class had to choose only one theme, as this was the only possibility of having representative samples of their schools. The seven classes chose different themes, like sexual education, drug abuse, tobacco, or music choices among teenagers.

The questionnaires they conceived allowed them to understand that to construct a balanced and well thought out questionnaire it is not enough to have some nice ideas. They discussed the different types of questions that could be used, their advantages and disadvantages, as well as how time consuming their treatment would be. So, these were very rich general discussions, and in some classes pupils' questions and comments already introduced notions like nominal, ordinal and interval scales, or some of the treatments that could follow the application of the questionnaires.

The next classes were dedicated to data treatment and analysis. Each group's members decided on their own what they would do in order to treat the data and to present their results both to their colleagues and to the educational community. This means that during these classes groups were carrying out diverse treatments, and discussing the choices that they should make. All groups began by using counting methods. But the strategies they applied were different, although many of them decided to go back to dyad work and do the counting two by two: one reading and the other one writing (like in Photos 1 and 2), thus changing the spatial occupancy, as they sat in a way that facilitated dyad work. Other groups still went on working all together, having one pupil who read the results and three others who wrote them down, also beginning to calculate absolute and relative frequencies.

Photos 1 and 2 – Counting and using dyad work, although assembled in 4-pupil groups

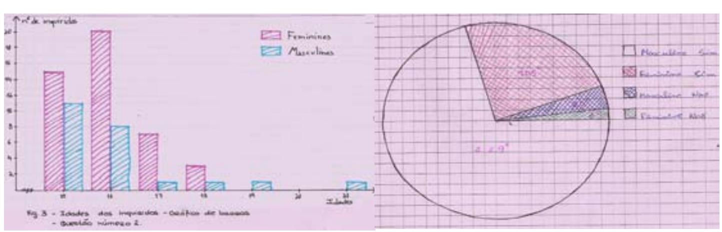
During the first two days of data treatment, pupils tended to use the statistical procedures and measures they already knew from the previous grades, like organizing the data in tables of absolute and relative frequencies, drawing graphs or calculating the mean. However, something was different from what is reported in other studies: they did not use data treatments in a merely procedural way, without discussing them, or why they should use them or not. Both videos, audio taped interactions, and observers' field notes illuminate a more reflective, relational and dialogical way of acting, reacting and working. Figure 1 illustrates how these tables of frequencies were organized:



6: Meios contraceptivos mais eficazes	Frequência absoluta	Frequência relativa
Preservativo	23	0,50 ou 50%
ñ. resp	23	0,50 ou 50%
Total	46	1,00 ou 100%

Figure 1 –Absolute and relative frequencies; percentages

Another interesting feature concerning data handling and treatment was that pupils really tried to be precise, but also visually appealing, when drawing their bar and pie graphs. Bar graphs included some of the most widespread social representations, like using blue for boys and pink for girls (see Figure 2). The pie graph was at first drawn roughly by hand, as they had no technical means to do otherwise (this was a poor school, with no easy access to computers). The next day, the pupils brought the necessary materials to produce a graph with the exact angle measures (see Figure 3), as they knew that drawing it without rigour would not allow them to present a nice piece of work to the educational community, nor to convince people attending their presentation about their findings.



Figures 2 and 3 – Bar and pie graphs

One of the first central tendency measures pupils calculated was the mean. This is a measure that has a strong social marking (Doise & Mugny, 1981) as pupils often use it even in academic life. Once again they did not only calculate the mean; they also discussed when this measure fitted their data better than the median, and they tried to be accurate. Empirical evidence shows that they did not experience some of the most common difficulties usually reported (Batanero, 2000). Figure 4 shows one of the means calculated in one group:

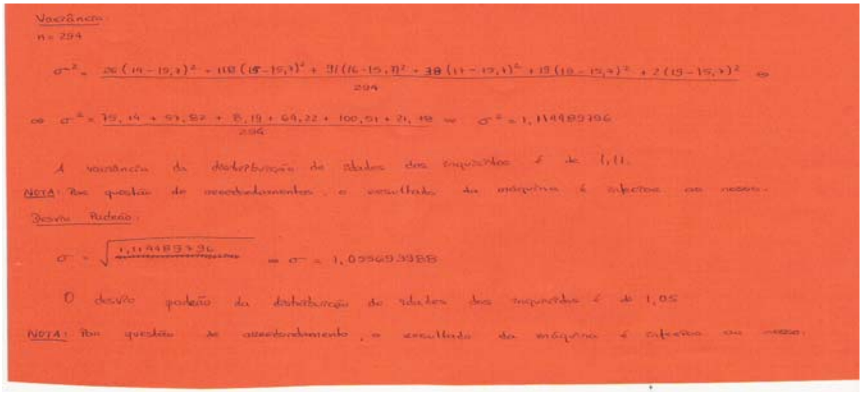


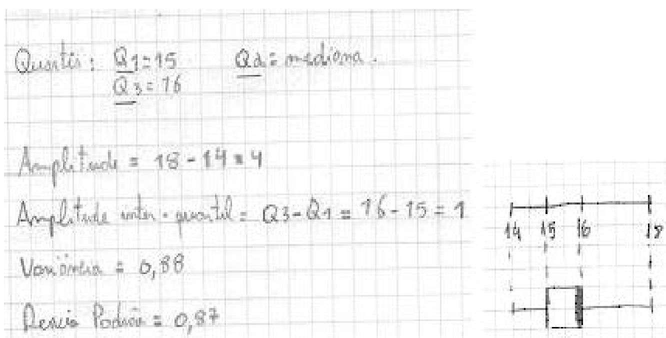
Figure 4 – Mean

Project work associated with collaborative work promoted great autonomy and sense of responsibility. Pupils were free to move around the room, to search information in text books and other available materials, to decide what they would do, to make their work progress. In many video excerpts, and in many photos, we can see the teachers sitting at their desk and taking notes about what is going on, while the pupils are working in their groups (see Photo 3). All the seven teachers, the three external observers, and the four external evaluators report that they had the impression that if the teacher left the room pupils would go on working as they were while he/she was there. The most curious of all this is that even teachers, who had worked with these classes for almost four months when they began project work, were astonished, and sometimes a bit uncomfortable with this situation. As one of them told us “I feel that they really don’t need me anymore!...”. And only after reflecting about what “not needing them” meant, they felt proud about themselves, and the work they had done, sometimes with classes that were really very troubled at the beginning of the school year. This empirical evidence illuminates the fundamental role of reflection upon practices, but also of collaborative practices among teachers/researchers. Looking also through others’ eyes allows for a different way of understanding the same episodes that we had already reflected upon ourselves.

Photo 3 – Pupils’ autonomy in project work

The 10th grade curriculum also includes some totally new statistical measures pupils must learn, like quartiles and standard deviation. As one of the main goals of project work is to promote pupils' autonomy, these contents were not presented by the teachers. They were learnt by the pupils themselves and were the last ones to be addressed by all groups. Different procedures were adopted by pupils within their groups: (1) some decided to read and compare the available textbooks, and then to discuss what they had learnt. Finally, they applied that knowledge to their data; (2) others decided that the pupils with higher marks in mathematics would read the textbooks, and then explain it to the rest of the group. Then they would all calculate these new measures; (3) in other groups, they all read the textbooks, then the pupils who usually experience more difficulties had to explain in their own words what they had learnt; then they all applied it to their data. But, what was common to all groups was that no one left behind these new concepts; all engaged in learning them, even when this meant a great effort and persistence.

Figures 5 and 6 illustrate the calculations about quartiles and their graphic representation, while Figure 7 is an example of the presentation of standard deviation.



Figures 5 and 6 – Quartiles

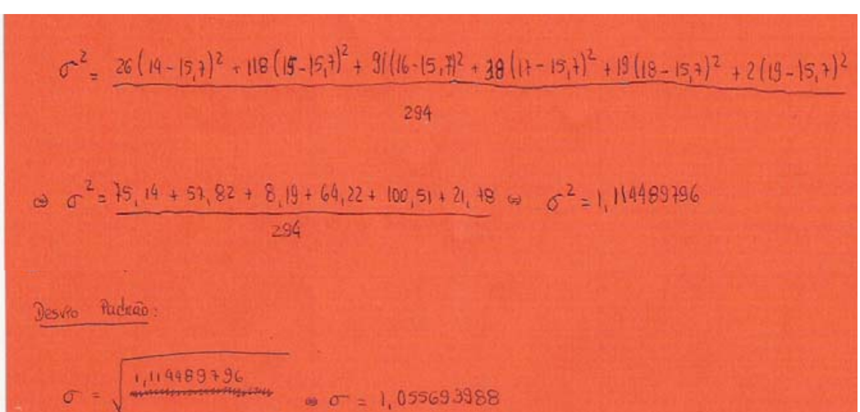


Figure 7 – Standard deviation

Pupils' answers in the questionnaires also illuminate some relevant features about collaborative work associated with project work in statistics. Sofia stresses that this type of work developed their ability to search for what they needed, to learn by themselves and to be autonomous: "(...) like this, we had to look for everything about statistics and learn and work with it without anyone's help".

Lúcia underlined the social competencies that were developed, like learning how to work in a group, and she states that this type of competency can also be useful in other subjects, that is, they are transversal competencies, which policy documents identify as some of the most important ones to be developed. As she accounts, "(...) we became more able to be part of a group and work within it. This is something that can also be used in other subjects".

João identifies another important issue: learning how to listen to others, and that sometimes we have to give up our opinions, conjectures, attitudes, ways of acting, because in order to work collaboratively we can no longer be always right, always convince the others, nor can we do only what we want to do without listening to the others. So, once again, he is stressing social competencies like deciding who is right, what should be done next, or when to argue and when to make concessions: "(...) it allowed us to listen to others' opinions and to learn to give in".

Pupils' answers and comments (questionnaires) support the same kind of empirical evidence that appear in the teachers' and external evaluators' reports, as well as the researchers' field notes, and videotapes. Although regarding collaborative work from different perspectives, according to their roles in the educational setting, all these participants report some common descriptions and interpretations, pointing out clear potentialities of collaborative work and project work for the promotion of statistical knowledge.

Final remarks

Peer interactions in statistics learning are an effective way of avoiding pupils' rejection of academic tasks and of developing their positive academic self-esteem, their ability to find solving strategies and to become more autonomous and critical learners. Collaborative practices allow for the creation of an atmosphere of respect towards diversity, as well as the desire to learn more. Thus, they play an essential role in promoting inclusive schooling principles (Ainscow, 1999; César, 2003), and avoiding exclusion, one of the great unsolved problems in the Portuguese educational system, namely in mathematics (César, 1994, 2003).

Pupils become legitimate participants of an educational community (Lave & Wenger, 1991). This type of learning experience promotes their autonomy and responsibility, having an impact on their life projects. It also facilitates teacher's better follow up of pupils' development and difficulties, as well as identifying and implementing adapted methods and strategies to develop pupils' competencies. Teachers also become more reflective about their practices, and more critical about the curriculum, facing it as a mediation tool to promote knowledge appropriation (Vygotsky, 1932/1978).

This project's results stress both the potentialities of collaborative work and the importance of promoting meaningful statistical tasks, like problem solving or project work, in which pupils choose the themes to explore, also plan and operationalize the different steps of their statistical work. Implementing interactive classroom practices was an effective way of promoting pupils' knowledge appropriation in statistics and probability.

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