From the mathematics classrooms: Dialogues and tasks under analysis. Returning to teacher autonomy

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Introduction

Teaching mathematics today is a real challenge. New learning theories, new ways to explain conceptual development, new forms of didactic work are issues being under continuous debate. Society, too, is in constant change, particularly with the advent of globalization, new multicultural arrangements of populations and new forms of inequalities. However, new methodologies are becoming available, as are new technological resources, thus promoting more sophisticated research methods through which we can obtain more precise data related to the analysis of classroom practices.

So, how should we educate mathematics teachers in order for them to be able to cope with so many changes in their professional lives and, at the same time, position themselves in a society in continuous movement? What approach should we use in the classroom that could be used as a model for future professional practices? Let us focus on this last question. When students take the different courses from a master's degree programme in mathematics education, they bring with them a long experience as students in school and, in some cases, also as a mathematics teacher. A natural tendency for teachers would be to *reproduce* the same teaching practices which they have experienced throughout all these years. Bourdieu and Passeron (1970) have studied these phenomena carefully and developed a theory of reproduction to explain in detail the mechanisms responsible for the constant reproduction of models identified in the school system. And in fact the natural tendency is to repeat these models to which teachers have been accustomed before.

If we are aware of this state of affairs which affects the educational system in a deep and consequential way, we can see how necessary it is to introduce *analytical perspectives* when working in classroom contexts. This has been one of the sources of motivation behind including a research project in our mathematics education masters' course at Agder University. The second one is related to and inspired by a modality of work established at this college for working in small groups since 1973. The idea is that after every lecture students are supposed to discuss the contents in *collaborative working groups* to increase understanding of the content matter (Cestari, 2000). A third source comes from the critical pedagogical work done by Freire (1991), Mellin-Olsen (1987), Alrø and Skovsmose (2002), and MacLaren and Silva (1993) promoting a constant *dialogical/critical position*. It is indeed this attentive attitude which helps teachers and students work in an engaged way and consequently with much more satisfaction and pleasure.

In this paper I would like to present a project called the Mathematics Education Research Group (MERG) where an inquiry/analytical modus is introduced among participants in the course *Teaching and Learning Mathematics*. Some research methods are also being implemented in order to give future teachers tools to analyse in a more systematic way the didactic situations in which they are working.

The aim of this presentation is to describe briefly the scope and main components of this project which has been developing now for 10 years, to introduce the main theoretical ideas on which it is based and to discuss some practical examples of what the students have produced.

Aims of MERG

1. to produce knowledge directly linked to the practice of teaching and learning;
2. to relate personal experiences in the learning and teaching of mathematics to theoretical questions presented in the scientific literature;
3. to analyse the activities related to the context where professional practices take place.

Components of MERG

1. Observation and analysis of mathematics lessons;
2. Task analysis (from the lesson).
3. Task based interview;
4. Seminar: paper presentation by the students

1. Observation and analysis of the lessons

The first approach to the data collected during mathematics lessons (in the following example in an eight-grade class), is through natural observation. Here an ethnographic perspective is taken: during the lesson the observer adopts a position trying to interfere as little as possible. These observations are registered on audio tapes and this year (2004), for the first time, video tapes have also been used. This audio and visual material is transcribed into protocols. This is an important step in the process of learning about constitution of meaning:

- the real time of the dialogue is extended
- identification of minimal interactive details: reaction time, silence, overlapping talk
- the characterization of the turns (questions, answers, explanations and so on)

In this way, two conversational features can be identified:

- the meaning of every turn
- the relationship among the turns

In fact as Goodwin and Goodwin (in Linell 1998) have pointed out, topic structure and participation framework are closely related. When we analyse conversations in this way, the mutual influence of participants can be clearly detected in the flux of the instructional discourse. Different approaches can be used in order to better understand the interaction of teacher students: discourse analysis (Coulthard, 1990), conversational analysis (Sacks, Schegloff and Jefferson, 1974), critical discourse analysis (Fairclough, 1995) and dialogical approaches to communication (Markova and Foppa, 1990 and Linell, 1998). In this project the latter approach has been chosen, particularly because it takes into account the sequentiality of the contributions. In a very pragmatic way the intertwining of meaning through the dialogues between participants can be clearly observed.

Below, two examples are given (Preiss, 2004). She has identified six components in a fragment of a dialogue from an eight-grade class when teacher and students are dealing with the concept of percentage:

- Problem (teacher)
- Explanation (student)
- Affirmation (teacher)
- Prompting (teacher)
- Solution (student)
- Response (teacher)

Turn	Time	Speaker	Utterance	Translation	Component
47	06:51	T	Dette var veldig lett for dere skjønner jeg. Hæ? Tim, tolv prosent av fire hundre kroner, hvor mye er det? Åss, assen løste du det da? Fortell meg det også gi meg svaret.	This is very easy for you, I see. Hmm? Tim, twelve per cent of four hundred kroner, how much is that? How, how did you solve that one, then? Tell me that and give me the answer.	PROBLEM
48	07:01	Tim	Øh, ti prosent av fire hundre kroner er førti	Erh, ten per cent of four hundred kroner is forty	EXPLANATION
49	07:04	T	Ja	Yes	AFFIRMATION, interrupting Tim's explanation and PROMPTING
50	07:05	Tim	= også en prosent er fire	= also one per cent is four	EXPLANATION
51	07:07	T	Ja	Yes	AFFIRMATION, interruption and PROMPTING again
52	07:07	Tim	= så tar du førti pluss to ganger fire, er førtiåtte.	= so you take forty plus two times four, that's forty-eight.	SOLUTION
53	07:16	T	Helt riktig.	Quite right.	RESPONSE

This fragment, at the beginning of the lesson - 6 min. 51 sec. - is constituted by seven turns and lasts 25 seconds. The time used by the teacher and the student is almost the same (11 seconds by the teacher and 14 by the student). This exchange can be characterized by one-to-one interaction in a public arena. The content refers to the concept of percentage introduced earlier in this lesson and is linked to the activity of selling objects at a lower price. The content of the discourse is related to mental operations and verbalized with natural and mathematical expressions. The posing of the problem by the teacher (47) is quite complex, involving the following elements:

- 12% of 400 kroner (the percentage of a specific amount of money)
- how much is that? (asking for the result)
- How did you solve that one, then? (the procedure used to find the solution)
- Tell me and give me the answer (the final solution)

The pupil answers in 3 turns (48, 50 and 52). The teacher affirms and prompts in between (49 and 51) and at the end of this dialogue he gives positive feedback, in response to the correct solution from the pupil. Quite typical in classrooms discourses is the confirmation of the answer of the students by the teacher even if sometimes the answer is not completed. This confirmation has a double function: giving a positive feedback, and, at the same time, prompting the continuation of the answer.

Line	Time	Speaker	Utterance	Translation	Component
127	09:48	T	Yes, på den første her, fjorten prosent, Mari, hva gjorde du da? Hva gjorde du?	Yes, in the first one here, fourteen per cent, Mari, what did you do then? What did you do?	PROBLEM
128	09:53	Mari	Jeg tok tre tusen fem hundre ganger en komma fjorten	I took three thousand five hundred times one point fourteen	EXPLANATION
129	09:58	T	Ja	Yes	AFFIRMATION confirming the answer and PROMPTING the next utterance.
130	09:59	Mari	=og det ble tre tusen ni hundre og nitti.	= and got three thousand nine hundred and ninety.	SOLUTION
131	10:04	T	Tre tusen ni hundre og nitti. Nå har varen steget fra tre tusen og fem til tre tusen ni hundre og nitti hvis du økte den med fjorten prosent.	Three thousand nine hundred and ninety. Now the item has risen from three thousand and five to three thousand nine hundred and ninety if you increased it by fourteen per cent.	RESPONSE confirming by repetition Mari's solution.

In this fragment of 16 seconds 5 turns of different types can be identified: in the first (127) the teacher states the percentage 14%, asks Mari for a way to do it, and immediately after, asks for a description. The pupil starts to say what she has done first (128): 3500 times 1.14; the teacher confirms it, prompting the next turn (129) when Mari (130) gives the solution: 3990. And then the teacher repeats Mari's solution and explains the increase in the price by 14 %. The interactional pattern established between teacher and student in this fragment is similar to the first example presented earlier.

So, what can students teachers learn from this analysis? First of all, they can learn how complex questions can be answered in a limited period of time. In this second fragment, for example, the question has four elements, including the solution and the approaches the pupil used in order to find the solution. Secondly, they can learn how there are elements of different types embedded in the same question. Thirdly, they can learn how pupils display the different reasoning procedures they use in order to find the solution to the problem, if they are asked to do so. The procedures used to solve the problem posed by the teacher became visible, thus offering possibilities for analysis, evaluation and, if necessary, re-examination (Cestari, Santagata and Hood, 2004).

2. Task analysis

The mathematical task occupies a central place in the dynamic structure of the lesson. It is constituted by multi-semiotic means, and presupposes specific knowledge in order to be solved.

According to O'Halloran (2004), meaning is made in mathematics in different check ways,

intra-semiotically through the grammars of:
- language
- visual images
- mathematical symbolism

inter-semiotically across these three semiotic resources which result in semantic equivalence or semiotic metaphor

So, the success of the resolution of a task in mathematics lies in the interaction of these three systems of meaning across the symbolism, visual display and language. It is indeed a critical factor in the meaning making, beyond that achieved via our linguistic and pictorial repertoires.

When pupils are confronted with written exercises related to, for example, the Pythagorean theorem, they are dealing with these three systems simultaneously: the explanation of the teacher (interiorized language), the geometrical figures and the symbolism (numbers, signs indicating operations).

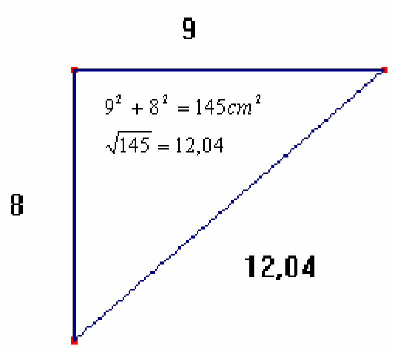
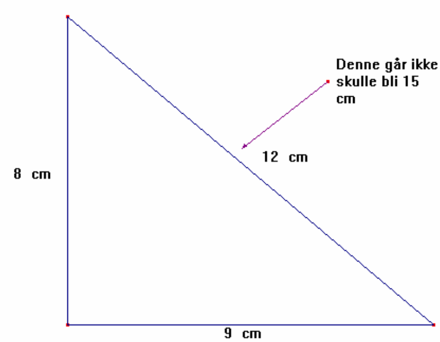
In the following task pupils initially read the instruction, which is composed of the following elements: a) the action they are supposed to do (check); b) the geometrical figure in which the three numbers are supposed to fit and c) the measurements of the sides (numbers and the unit of measurement, in this case, centimetres, written in an abbreviated form)

Check whether a right angle triangle can have these sides: 15 cm, 9 cm and 8 cm.

Six categories of solutions have been identified by Bueie (2004):

A) Comparison of lengths
B) Comparison of areas
C) Solution with the support of a figure

D) Solution through measurements
E) Only the answer
F) No answer

Category A: Comparison of lengths	*Category B: comparison of areas:*
$9^2 = 81\,cm^2$ $8^2 = 64\,\text{cm}^2$ $81 + 64 = 145\,\text{cm}^2$ $\sqrt{145} = 12{,}0\,\text{cm}$ Denne er ikke en rettvinklet trekant (This is not a right angle triangle)	$a)"cm"$ $= 9 \cdot 9 = 81$ $+\ 145$ X $8 \cdot 8 = 64$ $15 \cdot 15 = 175$
Category C: Solution with the support of a figure  	*Category D: Solution through measurements* 
Category E: only the answer Går ikke (No)	*Category F: No answer*

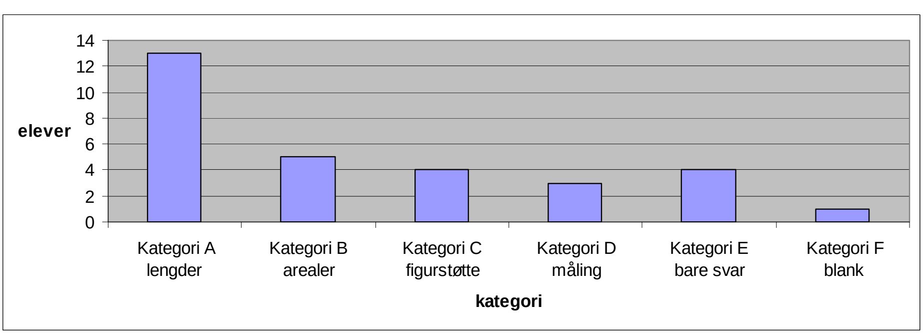


Figure 1: Number of responses for every category

What students can learn from this task analysis?

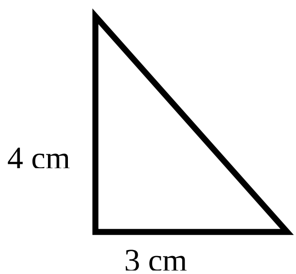
The main insight is related to the variation of the solutions. And this is an interesting point: it shows the different ways of understanding the nature of the task and gives the teacher elements with which to argue and challenge the actual state of understanding.

3. Task-based interview

A very pragmatic and insightful way to understand the reasoning of pupils is through the analysis of their solution to problems. According to Goldin (2000, p.520) "in comparison with conventional paper-and-pencil test-based methods, task based interviews make it possible to focus research attention more directly on the subjects' processes of addressing mathematical tasks, rather than just on patterns of correct and incorrect answers in the result they produce". The focus of questions formulated has been related to the following aspects: the topic of the task; the ways to solve it; the description of the strategies used to solve it and the justification of the solution.

This interview is based on a simple Pythagorean task done by three pupils: Ann, Carl and Tone. Ann and Tone have solved this exercise without any difficulty. Carl used a little more time and gave a wrong solution (he gave the answer as 25). Bueie (2004) focused the task interview on Carl.

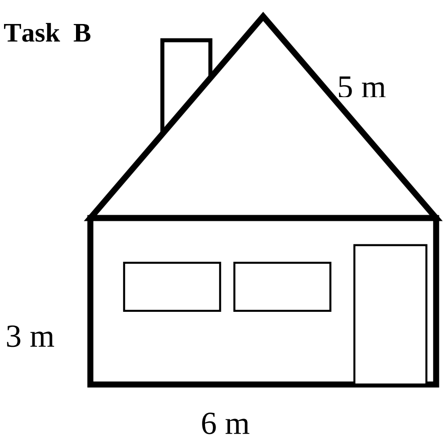
Task A



How long is the unknown side in this triangle?

Solution:

Task B



How high is this house?

Solution:

54. Carl: Oi, this maybe went a little wrong but…
55. Int.: This can go well…
56. Carl: Yes, I think it is 25 cm, but this is maybe… Hehe

57. Int.: Yes
58. Carl: I did what I think I should do, I took 4 times 4 and 3 times 3
59. Int.: Hmm. If you look now you found out that this side is 25 cm.
60. Carl: Yes
61. Int.: If you look at the lengths of the other two sides, what do you think about 25 centimeters?
62. Carl: Ehh. He he …I think that was quite wrong I do
63. Int.: Jaaa… in what way do you think?
64. Carl: He?
65. Int.: In what way do you think it is, too much, to little?
66. Carl: I think it's too much. I think there's one more thing I must do. I think it's something to do with the square root of 25. Yes.
67. Int.: Mmm Yes
68. Carl: Yes, I think that was it, but I m not sure.

When Carl was to explain why he has done the calculation in this way his explanation was like this:

90. Carl: First I took 4 times 4
91. Int.: Yes
92. Carl: So I got 16, then I got 3 times 3 which is … so I got 9 plus 16 and so this makes 25 and then I took the square root of this
93. Int.: Do you remember how much the square root of 25 is, or…?
94. Carl: 5
95. Int.: Hmm
96. Carl: Ja
97. Int.: When you write 4 times 4 why, why do you do that?
98. Carl: He… tja. No, it's just that, that I know we should do this
99. Int.: Yes
100. Carl: I don't know why I do that, but I only know that I'm supposed to do that, because that was the way the teacher said we should do it.

The comment bellow is made by the student who interviewed Carl:

"As regards the question related of the justification to what he has done and to the answer "it was this way the teacher said we should do it"… I conclude that Carl has a very mechanical relationship with Pythagorian theorem and how to use it." (in Bueie, 2004, p. 16, the translation from Norwegian is by from the author).

The distinction between conceptual and mechanical ways of dealing with the solution of a geometrical problem is an important perspective for teachers when analysing the work of students. To learn mathematics means also to be able to understand how concepts are interrelated.

At the end of the interview another aspect to be mentioned emerges: the dependence of the student on the models offered by the teacher. It seems that the student has not been able to understand and appropriate the explanation about how to find the length of the hypotenuse. Mellin-Olsen (1991) uses the expression "control of knowledge" to indicate the possibility of the pupil using his own knowledge and his own ways of understanding independently of the teacher.

What can students teachers learn from these task-based interviews?
The main issue is to grasp where the problem for the pupil is located. What kind of conceptual relation is missing or is not very clear? It is interesting to observe that in the example given above, the confusion lay exactly in the addition of the square of the sides related to the square-root of the hypotenuse. In fact, this link between the addition of the square of the sides and the square of the hypotenuse is still missing. The ways to network these isolated elements in the solution can emerge from the identification of these missing links. Certainly the teacher has more effective resources to explain the nature of the solution of a problem when s/he knows exactly where the pupil misses the link.

4. Paper / Seminar
After students have collected and analysed the data from the lessons they start to put together and write a research paper. The importance of this stage is the integration of the different kinds of data, which consequently gives a deeper grasp and understanding also of the phenomena occurring during a math lesson. One of the important elements here is the review of questions from the scientific literature and the discussion of their own findings. In this way they have the opportunity to amplify their knowledge related to their own research question contrasting this with research questions about the same topic formulated in published papers.

The presentation of the paper in a seminar is the next step; here it is necessary to select relevant information from the paper and present it in a clear and coherent way, using the technological resources available today. This year all students used Power Point for their presentations. Immediately after, they get feedback from colleagues and from the teacher, the coordinator of the project.

Final remarks

- Taking an analytical position related to the professional context promotes the possibility of new perspectives for understanding and action. The classroom, from the *context* of teaching-learning, became, in this way, an *object of study*. Possibilities and opportunities for necessary changes can be identified easily. The next step, the implementation of changes, is elaborated from insights emerging from students' own analysis. Here the value of micro-analysis lies in understanding the complexities of the mathematics lessons and, at the same time regarding it as a potential resource for transformation.

- The increasing sense of autonomy provoked by this analysis can stimulate the same sense in the pupils. Only when we navigate in a mental space with our own sense of orientation we can understand what intellectual independency means. Then we can control our knowledge and become more autonomous. This aspect has important political implications: it stimulates independence and creativity.

- It is during the experience of the micro-analysis of didactic situations that new ways to present problems, to explain, and to give feed-back can emerge. The focus on the dialogues from the lessons, on the analysis of the tasks, open new mental spaces for understanding and consequently new areas of intellectual and emotional pleasure which stimulate so much the desire to understand mathematics.

- New theoretical and methodological directions: the *multi-semiotic* approach to the analysis of classroom activities and the m*ulti-modal* way to present data from the context of the classroom.

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