School mathematics as a major subject for 'humanistic education'

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Introduction

One of the major problems of today's school mathematics, which is one of the core subjects in compulsory education, is the issue of 'mathematics for all'. What is the purpose of mathematics education as a national mass education? Why is it that we are now making such an effort to teach mathematics to all students? Without question, mathematics is needed in the development of scientific technology. However, not everyone is going to be a scientist or a technologist, and it is difficult to claim that most of what we teach in mathematics class is needed for leading the present-day life.

Mathematics has occupied an important place as a core subject in school education ever since Ancient Greece. This shows that teaching and learning mathematical knowledge has been considered 'educationally' important beyond the practical usefulness of mathematics. Why is mathematics 'educationally' important?

As human-beings we live at two levels of life; the practical level and the theoretical level, and the shapes of living and education vary according to the level on which we put more weight. Attaching great importance to theoretical knowledge is the tradition of the Greek type of education for the free citizen and the medieval seven liberal arts education in Europe led by the scholastics. This was also true in the traditional Korean noblemen's education which valued the liberal education based on the Four Books and the Three Classics of Ancient China. But unfortunately our ancestors undervalued arithmetic as only a practical tool, and the noblemen had no concern for it. This is a big difference of the civilizations of the Orient and the Occident. The English word 'theoretical' was derived from the Greek word *'theoria'* which means 'seeing', while 'practical' was derived from the Greek word *'praxis'* which means 'doing'. The education for theoretical knowledge is the tradition of education for developing conceptual means, or our mind's eye, for seeing the phenomena (Lee Hong Woo, 2002).

From the period of Ancient Greece until now, if there is anything that has not changed in school mathematics, it will be the fact that theoretical knowledge which goes beyond practical knowledge and is not very much motivated with practical problem solving, has continued to be taught. The practice of mathematics teaching is as it used to be, but, especially entering the 20th century the pragmatist point of view that regards mathematics as a system of problem solving tools, which is supported by Dewey's experientialist philosophy, was emphasized, and since 1980s the endeavor for teaching practical knowledge, such as problem-solving and application of mathematics, has been continued.

According to the examination by Niss (1996), although it is difficult to judge that the practice of education was directed in specific ways, in the history of mathematics education of 20th century we can observe an overall oscillation between two kinds of goals of mathematics teaching; one is the disciplinary goal to further the development of reasoning and logical thinking, concentration, persistence, and intellectual functions; the other is the utilitarian goal. And we can also find cultural or aesthetic

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aims such as the appreciation of beauty in the geometrical forms, ideals of perfection in precise logical reasoning, and admiration for power of abstract thinking. But, the question about the extent to which we realize how much the fundamental goal of education is to cultivate the “internal eye” to 'see' the phenomena, and to foster virtue, humanity and disposition of the students through the mathematics curriculum of each country of the world, has to be raised.

Despite of every endeavor to reform the curriculum, the practice of mathematics education that is centered around theoretical knowledge and the teacher taking the lead, has scarcely been changed (Bodin and Capponi, 1996). On the whole the goal to develop the ability to solve real life problems does not match well with the character of the topics that have actually been taught in mathematics classes. There are still few items of practical knowledge serving to efficiently solve real life problems present in school mathematics, except at the elementary school level. We as mathematics educators should face the actuality of mathematics education and re-examine the educational value of theoretical mathematics knowledge.

Here, investigating the roots of thought in mathematics education, we will maintain that one of the major ideals of traditional mathematics education based on theoretical knowledge-centered education is the cultivation of humanity. So we explore the way today's mathematical pedagogy should first turn to in order to realize this ideal.

The roots of humanistic mathematics education - I

Platonism

In the Western civilization which has its roots in the ancient Greek civilization, mathematics was traditionally located at the core of education. If we trace back the Western ideological history to Greece, we can infer the real reason for learning mathematics. It is well known that Plato said that those who have not learned mathematics (geometry) cannot be called educated. Why did he said that? Plato's view on the nature of mathematical knowledge and its educational importance can be summarized as follows (Yim Jae Hoon, 1998).

Plato assumed reality to a metaphysical world called Idea. In his dialogues *The Republic* and *Phaedo*, Plato thought that the world of ideas and the physical world are connected as 'up and down' and that the upper part reveals the essence of the lower part in its purest form and that the lower part embodies the essence of the upper part. According to Plato, people cannot lead a humane life because of body lust, the limitation of the sense and ignorance. But man is a being for whom it is not good enough to remain in this state of life. He has to overcome it in order to be a real human being.

To Plato, education is a way to make people turn their eyes from the real world, like in the famous cave (in The Republic), to the true world, the world of ideas. To him, an educated person is someone who perceives the world of reality by stepping over the shadows of the real world, someone who has recovered the pure spirit inside by freeing him/herself from the limitations and confinements of the body.

Plato cited arithmetic as the first useful subject to lead the spirit to the reality, and after arithmetic he cited in The Republic geometry as the second such subject. As mathematics studies, in a logical manner, the invisible archetypes of things by using the *eikon* that is incompletely embodying it, mathematics becomes the ladder for climbing to the "upper world" from the "lower world." By giving us the clue to understand the harmony and beauty of the world of ideas which exists at the other side of the visible world, mathematics enables the human being to get ready for having his

or her mind being led from 'the world of senses' to 'the intelligible world' and by providing such awareness, mathematics becomes a subject of educational value.

In this way, to the free citizens of Greece mathematics was the most appropriate subject to open their 'eye of the soul' and turn toward the world of ideal reality. So for them learning mathematics was a necessary stage in 'the journey toward truth'. Thus a man who did not know arithmetic and geometry was not considered capable of seeing the image of truth; his 'eye of the soul' to see the truth was not opened. In other words, mathematics is a discipline that started as a study to foster real human education.

It is interesting to examine how this conception of mathematics education has been pursued in Western secondary school education. I think that, today, as mathematics has become a subject required for all free citizens, the ideals of Greek mathematics education have surpassed the dimension of elite education and become an ideal of public mathematics education.

The structure of knowledge and the 'reality'

The Greek educational ideal pursuing 'the world of reality' was developed into a new dimension by Bruner who wanted to make students see the 'structure of knowledge' through the *spiral curriculum* and *discovery learning* in the middle of the 20th century.

According to the structure-centered curriculum, the content of a curriculum should be the structure of knowledge implicit in the outer layers of knowledge. Bruner used the structure of knowledge as a synonym for 'general ideas that constitute the foundation of the subject', 'basic notions' and 'general principles', which means 'the lens through which to look at matters' or 'the ways of thinking' that define each subject. But it is not a state we can actually reach or realize, but rather a standard for looking at matters that we should figure out or try to reach. Like *Idea*, knowledge is such a thing that is impossible to reach completely, so the harder we try to pursue it the more we realize that we are ignorant.

We can understand this by considering, for example, how difficult it is to grasp the meaning of the notion of function, functional thinking or group in spite of much discussion and effort.

Bruner thought that his structure-centered view of the school subjects is different from Dewey's pragmatist view of them and said that Dewey's mistake was that he looked at the school subjects mainly from practical perspectives. It should be noticed that the discovery learning-teaching method that Bruner suggests is a theoretical combination of Plato and Piaget by dynamically adapting modern developmental theory combined with classical rationalism (Shulman, 1970). The structure-based curriculum shown in Bruner's *The Process of Education* (1963) can be seen as an effort to face the 'structure of knowledge' or 'the form of knowledge' by means of the spiral curriculum and discovery learning, in accordance with the *Zeitgeist* of his days which was called structuralism. We can say that this was an attempt to successfully implement the education on theoretical knowledge in order to cultivate the internal eye 'seeing' the real nature of phenomena, and now such an ideal is being pursued as being usual.

The roots of humanistic mathematics education - II

The practical mathematics of Archimedes and nature's mathematical order

There is not only one root of thought in mathematics education. In fact there are at least two. A second root is the practical mathematics of Archimedes, Heron and others, and the rise of the scientific investigation of the mathematical laws of nature,

as was initiated by Galileo. Practical mathematics has the spirit of inquiry, discovery, experiment, calculation and application, and has become the essential tool for the inquiry into nature since Newton. After that the basic character of nature as governed by the physical laws described by mathematics began to become known.

> "The leading figures of the Modern Era - Kepler, Galileo, Descartes, Newton and others - emphasized in the most elevated terms the Pythagorean idea of the divine mathematicalness of the world. To quote Leibniz : 'Cum Deus calculat, fit mundus' (As God calculates so the world is made). ... Hilbert spoke of the 'pre-established harmony' between mathematics and physics. ... Einstein returned to the thought of this harmony, but this time he spoke of the pre-established harmony between physical realty and mathematical structures ... He used the example of conic sections, realized in the orbits of celestial bodies, to explain the sense of this harmony..." (Vizgin, p. 265-267)

The awe-inspiring nature of the mathematical knowledge which constitutes the order of the universe, obviously demands that mathematics becomes a subject for the moral education of the students, going beyond practical tools for solving real life problems. Fröbel was an educational thinker who highly appreciated the value of mathematics for humanistic education with the ideological background that the harmony of mathematics and the physical world is the proof of the existence of God. He investigated the problems in the nature of human education and made the following assertions (Han Dae Hee, 2000).

Through mathematics a man can conceive the divinity that exists inside the universe, both human and natural. Through studying mathematics, he comes to know that there is a mathematical order inside the natural world, and that this order is expressed through the laws of speculation by the pure human spirit. Here, he becomes conscious of the divinity inside man and nature, and becomes a valuable person who, by realizing the spirit of God, believes in the existence of God and lives by the will of God. So, to Fröbel mathematics can and must be an educational means to perceive the spirit of God in nature as well as in man and it should be a subject that has essential meaning in 'human education'.

To learn mathematics is to acquire the eye to 'see' the world as created by God through the 'form' of mathematics, which means that our minds of us change accordingly and we can become pious human beings. Thus, to Fröbel, "education without mathematics is weak, imperfect patchwork" (Fröbel, p. 208). The nature of mathematical knowledge strongly demands school mathematics to become a subject for humanistic education going beyond practical usefulness.

Computational mathematics and the sense of truth

The mathematics developed in the cultural circles of India and the Arab world after the decline of Greek mathematics was a practical mathematics which centered on computations. With the discovery of the decimal notational system, the mathematics of India and the Arab world brought the development of the four arithmetical computation methods which greatly influenced the development of human civilization. Also, the development of algebra, centering on solving equations, and the introduction of the algebraic symbols by Viète, brought the development of modern mathematics.

Formulae for computation, represented by letter variables, are algorithms deduced from basic rules, and are an amazing mathematical form of seeking truth, where the discovery of an answer and its proof are conducted together. If we do not feel 'the

computational spirit and the sense of truth (*Rechnungsgeist und Wahrheitsinn*)' as Pestalozzi said, when we look at the formula for the roots of quadratic equations, or the fundamental theorem of calculus, we do not 'see' the true form of mathematics.

The two branches of mathematics - geometrical mathematics and computational mathematics – were combined and brought the birth of Descartes' analytic geometry in modern Europe. And this again, combined with applied mathematics, led Newton and Leibniz to invent differential and integral calculus which provided the basis for modern scientific civilization. The computational mathematics which attracted Descartes, who sought for the universal method to solve problems, and Leibniz, who even said that "As God calculates so the world is made", is the most certain method, together with axiomatic mathematics, of inquiring truth. In the middle of the 20th century human beings invented the computer which led to what is called the information society and moved on even to dreaming of 'computopia'.

The spirit of computational mathematics ought to be the base of national education, making students pursue the sense of truth and become an individual of virtue. We should teach the students to realize that the method of computation is by no means a mechanical operation but rather the most wonderful method which human beings have created to uncover the truth.

The roots of humanistic mathematics education - III

One other root of thought in mathematics education can be found in the formal discipline theory. The basic assumption of this theory is that general mental ability such as inference, memory, imagination and will power, which are 'the muscles of the mind', can be trained by drilling, just as the muscles in our body. Once trained, they can be used for solving many intellectual and practical problems. It is claimed that the way to train such general mental abilities is to study traditionally acknowledged subjects such as classical language, history, mathematics, science, and others (Lee Hong Woo, 1992).

This theory has been denied by Thorndike's Identical Element Theory. According to Thorndike, transfer is not guaranteed by 'general ability', but by the 'identity of elements' between two situations, only to the extent that there is sameness. Dewey (1960) criticized the point of view that if one learns mathematics, the inherent logical thinking forms are acquired and one can then think logically in any domain, and stated that 'logical form' and 'logical thinking' should be distinguished and that the 'logical' thinking of a child should be gradually trained through his own reflective thinking, rational thinking, and proper thinking.

But, according to Vygotsky(1965) the formal discipline theory is not related to the training of specific skills, but related to education which activates the dimension of consciousness, including awareness, abstraction and self-control, such as mathematics and classical language education, And mathematical thinking is the result of the introduction to the mathematical forms of knowledge, and learning mathematics is the means to see the world through these forms of knowledge and to change the minds of students. Education for theoretical knowledge can have educational value in that through such change of mind people can become righteous human beings.

Though the formal discipline theory has been criticized and refuted by Dewey, Thorndike and others, if viewed in the light that emphasizes the form or structure of knowledge, reflective thinking, and the activation of the domain of awareness and consciousness, it is as certain as ever that learning mathematics 'properly' changes one's 'way of looking at the world', and cultivates one's 'logical thinking and reasoning ability'.

Foundation of humanistic mathematical education

The above mentioned thoughts of traditional mathematics education require that mathematics education makes students see the image of truth and investigate the order of nature by means of the computational methods of seeking the truth and by intelligent logical thinking. A great educator who set up mathematics education as humanistic discipline was Pestalozzi.

The educational thoughts of Pestalozzi are called the idea of basic cultivation. He tried to cultivate in a natural manner the three basic powers of man - mental power, moral power and functional power - and wanted to realize the development of 'a power of higher humanity' through mathematical education. He believed in the educational value of mathematics as the stepping stone to cultivating the mind.

He made the following claims. To learn mathematics is to do a 'spiritual gymnastics (*Geistesgymnastik*),' 'To divide the spirit of computation and the sense of truth is to divide what God has combined (*Wer Rechnungsgeist und Wahheitsinn trennt, der trennt, was Gott zusammen gefügt*)'. 'To compute and to count is the basis for every order of the brain (*Zählen und Rechnen ist der Grund aller Ordnung der Kopf*),'Mathematics is the operation which raises 'the natural aptitude of reason to the power of reason (*die Vernunftanlage des Menschen zur Vernunftkraft*),' Mathematics is the 'basic educational gymnastics (*Erziehungsgymnastik*)' (Kim Jeong Whan, 1970).

I think that such thoughts on mathematical education has been pursued consistently in the West, even though almost implicitly, by mathematics educators who led reform movements of mathematics education. In the early 20th century, Perry (1902) in his speech "The Teaching of Mathematics" at the British Royal Society, suggested eight kinds of usefulness resulting from the study of mathematics. The first three are producing high emotions and giving mental pleasure, brain development and producing logical ways of thinking, and the aid provided by mathematical weapons in the study of physical science. Let's focus on the order here. It should be noted here that between the intrinsic and extrinsic purpose of education, and between the value of knowledge itself and its instrumental value, the first one, an affluent mental life, intellectual satisfaction and seeking for beauty are given the first priority.

It is sometimes being said that actuality is the parody of an ideal. As in other areas, the actual state of affairs and the ideals are different in mathematics education, too. Unlike the Greek ideology that mathematical education leads the eye of reason to reality, mathematical education has drifted towards formal instruction. Rather than gaining the sense of truth and spiritual happiness, it is not the case that the more mathematics students learn, the more do they come to loathe mathematics?

The danger of mathematical education

Let us take a look at the preface of the book written by Dewey and McLellan (1895); *The Psychology of Number and its Applications to Methods of Teaching Arithmetic*. It says; "There is no subject taught that is more dangerous to the pupil (than mathematics) in the way of deadening his mind and arresting its development, if bad methods are used."What is the reason why Euclid makes the minds of the students suffocate as Descartes said? How can we teach mathematics so that the students understand the true meaning and taste of mathematics so that their human nature can be cultivated? This will be the most important problem that we, as mathematics educators, must face.

Piaget's theory according to which thinking is operation and operation is internal reversible action, calls for active learning principles. Polya (1965) calls for a teaching atmosphere where guessing, discovery and desirable habits of thinking induced by the questions and suggestions of the teacher as a midwife, are emphasized. Dewey (1960) sees reflective thinking as having an important role in solving practical problems as well as in the development of logical thinking, so this gives a central role to reflective thinking in education.

What does 'doing mathematics' mean? Let us look at Freudenthal's claims (1983): To do mathematics is to observe one's own or others' mathematical activity and to reflect on it. To him, the main body of mathematical activity is reflective thinking.

Now what is teachable? The following claim by Gattegno (1963) is meaningful. 'Only awareness is teachable.' Awareness can be interpreted as realization. Man becomes educated only by realization. Then, how is awareness formed? The conclusion given to us is that it is obtained by activity, discussion and reflection. Let us learn from the following saying from the Analects of Confucius. "Mere reading without thinking causes credulity; mere thinking without reading results in perplexities (學而不思則罔 思而不學則殆)."

Mathematical education for every student should be conducted as cultural education in the real sense. Mathematics is the most pertinent subject to educate people to lead a life toward the reality by opening their 'eyes of awareness'. Here, mathematical activity and reflective thinking play major roles. As Pólya (1965) points out, "What the teacher says is not unimportant but what the student thinks is a thousand times more important."

The didactical analysis of the structure of mathematical knowledge

If we want to make a change, through mathematical education, in the way students view the world and the way they live their lives, , and if we want to realize humanistic education through mathematical education, first of all, there should be changes in the mind of the mathematics teacher and an educational philosophy of the teacher should be established. But, as Pólya (1965) emphasized, it is impossible for people to teach what they don't know. A deep understanding of school mathematics on the part of the mathematics teachers is the alpha and omega of mathematical education.

The point of view which emphasizes 'the structure of knowledge' or 'the forms of knowledge' is to prescribe the meaning of education for knowledge of the nature of this knowledge. But, school mathematics is a formal 'closed' kind of knowledge that does not reveal its nature. To 'open' this up and to realize it into educationally meaningful knowledge through didactical analysis, i.e. mathematical, epistemological, historio-genetic, psychological, linguistical, practical and educational analysis of the structure of school mathematics, may be the most important task for the teaching of mathematics for humanistic education.

The noteworthy elements here are the historio-genetic developments of the school mathematics of Clairaut, Branford, Toeplitz and others, and *Didactical Phenomenology of Mathematical Structures* by Freudenthal. These are all thought to have their goal in trying to open the closed school mathematics up so as to make it become didactically meaningful knowledge. This task, as Freudenthal insists, has to be the starting point for mathematics education research. The didactic phenomenological analysis of ratio and proportion, of group, function and others concepts that Freudenthal attempted, and the didactical analysis of the probability concept that Kapadia and Borovenik (1991) attempted, show how hard it is to 'see' the deep idea, the structure, the form and the nature of mathematical knowledge. How could a

teacher who does not see it properly, or perhaps does not even try to see it, think about teaching it? The research mentioned shows how the mathematical knowledge that we are teaching and learning is only at the surface of knowledge and how we are missing the 'educationally' much more important essential viewpoints. Mathematics education research ought to start, first of all, from inquiry into the deep 'structure' of school mathematics.

And we should also not undervalue the role of the oral and written language in education. Recently discovery and constructivist approaches based on learner-centered activities using the concrete materials, are emphasized, but it should not be overlooked that the language used by the teacher in education plays a more important role than does anything else. The Socratic “obstetrics” in the Plato's Dialogues, the Analects of Confucius and Pólya's modern heuristics are written in a dialogical style centered on the teacher. And the sentences in which knowledge is recorded may be a kind of residue of the realization, as satirized in a Chinese classic, but it is the only one clue which we have in guessing what the realization is. Thus we can do nothing but try to get the realization by means of the written sentence.

Concluding remarks

Now the time has come to rediscover the idea of mathematical education as public education and as cultural education, and to recover the original features of mathematical education.

Today it has become one of the central themes of mathematical education to develop the student's power to solve practical problems and to apply mathematics, but generally this does not fit with the nature of the mathematical knowledge which we are actually teaching in the school. Here we have noticed this fact and have re-examined the idea of traditional mathematics education centered on the fostering of theoretical knowledge.

Education is an endeavor to identify and realize ‘the forms of living’ trying to approach the supreme good, i.e. human virtue. The great thinkers in the Orient and the Occident almost without exception recognized the world of logic, forms, substance and reality beyond the world of facts, phenomena and mortals. Teaching for theoretical knowledge is to make the students 'see' the phenomena through such forms and at the same time to implant the belief that it is a valuable life to live seeing the phenomena, so that students accept the theoretical posture of living, devoting themselves to endless inquiry in order for them to eventually see ‘the forms of living’ (Lee Hong Woo, 2002).

Here we ought to notice that coming to the present day, the tendency in which the basic physical laws are described in the language of mathematics has become more clear, and the quasi-religious beliefs of great mathematicians and physicists, such as Hilbert and Einstein, about the mathematical nature of the physical world have become deeper.

Mathematics reflects the realty - 'the logical cause of the universe' - and we face the image of it when we learn mathematics. The main purpose of teaching and learning the domain of knowledge called mathematics is to go beyond practicality and achieve the humanistic education that help people come to realize the existence of the reality that rules the world of phenomenon, and to cause endless longing for it.

Is this not the idea that school mathematics as a major subject for humanistic education ought to try to attain? If the pendulum of mathematical education so far seems to have swung in the direction of ‘Satan’ trying to pull for 'formal' meaningless education, we have to continuously push the pendulum in the opposite direction.

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