The design of practising in mathematic textbooks

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Why discuss this topic?
As everybody knows, mathematics is a systemic and abstract subject with special rules. It is difficult to imagine anyone mastering mathematics without enough practice. However, inappropriate exercises may hurt the enthusiasm of students. So when a student learns mathematics, how many exercises does he or she need to do, and how does (s)he to do them? That is a big problem to us and has been for a long time.

Typically, practising should be arranged throughout the entire process of mathematics learning. When a student learns new mathematical contents, he or she may need to draw diagrams, calculate data, and estimate the corresponding results. Then, by investigating the concept, student can understand the new knowledge. And then, to promote student's deep understanding and mastery of the material, a great deal of practice work is needed. Students need necessary adjustments to clear up their thinking, and to consolidate what they have learnt by completing similar exercises and varied exercises, exploring counter examples, so as to make themselves proficient with the presented material they just learnt.

At this moment, I should like to say, in Western and Eastern countries mathematic educators have different opinions about how much practising is necessary to master mathematical concepts. It seems to be an unmanageable problem.

In Western countries, some educators believe that several repeated exercises are unnecessary and that a student must learn by understanding. I agree with this idea, but how does an educator make a student understand?

In China, many people believe that the mastery of a branch of knowledge is impossible without a certain amount of exercises. There is a Chinese saying "Shu nen shen qiao," which means, "Practice makes perfect" or "skill comes from practising". Well then,

- Do we need practising in mathematics learning?
- Does practising create understanding?
- Does practising make perfect?
- Does practising make students bored?
- Does practising create burdens for students?

For these questions, it is difficult to answer with a *yes* or *no*.

In recent years of curriculum reform in China, the new mathematical curriculum standards emphasize the idea that students must actively study and "learn by doing". However, in China, in some academic research a new trend that ignore exercises has appeared. This trend considers mathematical exercises as passive learning, simply consisting of imitation, or rote learning. According to this philosophy, lately some teachers do not dare to expose students to a sufficient volume of exercises in their teaching. "Learning by doing" makes students actively investigate mathematics in real problems to understand what mathematics is and where it comes from. Yet, this notion is,merely taking us from one extreme to another.

According to the present situation of foundational education in China, the textbook

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is playing a very important role in mathematics education, and is greatly effecting students' achievements. The textbook is the main and the most important resource for carrying out instruction to achieve the curriculum goals, and it is the most significant media for students' cognition and development.

As a national professional textbook developing and publishing unit in China, we, at People's Education Press (PEP), have performed research and compiled school textbooks for more than 50 years. For doing mathematical exercises is an important and debated topic, and designing appropriate and helpful practices was always our major resource. So at this moment, I would like to share with you some ideas on how to deal with exercises in mathematics teaching and how to design appropriate practices in mathematics textbooks.

The necessity of practising in mathematics learning

It is very important for students to actively study and develop their own understanding. And by doing a number of exercises, students can gain further deep understanding and mastery of mathematics. So, we need to investigate both these ideas, to get rid of the disadvantages we have experienced, to combine the advantages in our teaching, so as to advance our mathematics education into a higher level.

About the necessity of practising, I would like to explain three points:

(1) The views of practice

(2) Consolidation and accumulation of knowledge(3) Formalization and normalization of knowledge.

The views of practice

We know that knowledge originates in practice, i.e. knowledge comes from practising. So when a student learns mathematics, he must engage in the real practical activities to obtain experiences. These practical experiences enable the student to understand what he is studying, what mathematics is, and how it is.

Then, the goal of having practical experiences is to help students abstract mathematics from the real world, to help student develop their thinking ability, rather than to just let student solve the real problem in their construction. Some people believe that studying mathematics means researching and solving practical problems in reality, and that exercises in textbooks will only carry students away from reality, making them fall into tedious and formalized mathematics theories.

I believe that studying real-world mathematics would help students to understand the material. However, it is impossible for students to study real problems all the time. In the practical activity stage students just have the knowledge of mathematical facts. But they should study further. In such further study, students cannot always dwell on the reality. We need to take the concepts of mathematics and study their nature and the general rules.

In other words, the mathematical study should start with students' participation in real-world experiments. Students can learn the origins of numbers, characteristics of shapes, and the relationship and rules among different quantities, numbers and shapes under proper conditions. However, intuitive understanding and initial feeling for mathematical concepts are not enough. They just lead to the knowing of facts. Students need to think about the subject carefully and abstract the mathematics result from the objective fact. And having abstracted mathematics from reality, students should go further with their investigations, to master the essence of mathematics and its general rules.

Within these studies, students need to practice the mathematics materials, to be able to think, to feel, to completely understand the materials. In this stage, in some sense, doing exercises is also a kind of practice, or should we say “learning in practice”. The student not only practices with practical but also with conceptual mathematics problems.mSome Chinese mathematician call conceptual practice “cooking the middle of the fish”, which means to pay more attention to the middle part of mathematics (the conceptual theory). Modern mathematics curriculum reform often seem to be in opposition to this philosophy. They suppose that students cannot understand mathematics without being exposed to its contextual uses.

In my opinion, we should find a proper balance on this problem. A fish without a head and tail is no longer a fish, which means mathematics without its applications is no longer truly mathematics. However, if you cannot cook the most important part of the fish well, you will never have a delicious dish. Likewise, without mastering the essential mathematical theory, a student can never solve real world problems either.

I believe that when the exercises are meaningful, valuable, well-chosen, and well-organized, students can achieve a greater understanding of mathematics by doing them. Only through such integrated study, can students grasp and apply mathematics successfully. It is a very one-sided notion to consider exercises as nothing more than a mechanical discipline, dealing with meaningless problems with no real world application.

At PEP, in our textbooks, after introducing every new concept or method in mathematics, we always design a specific number of exercises.

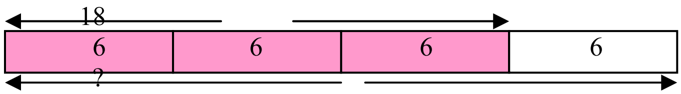
Example,
When teaching how to divide by a fraction, we present this problem:

$18 \div \frac{3}{4}$.

We remind students to recall the reasoning of whole numbers division, which is the reverse process of multiplication, and to think out this division.

With the following illustration, students think that this requires us to find a number, which can be multiplied with $\frac{3}{4}$ to get 18 as a product. In other words, when this number is divided into 4 equal parts, the 18 takes up 3 of the 4 sections.

So each section is 6 (for 18÷3 = 6). So the number must be 6 times 4, that is 24.



Because 18 divided by $\frac{3}{4}$ is equivalent to (18÷3)×4, and we know that (18÷3)×4 = $18\times\frac{4}{3}$. So, $18\div\frac{3}{4} = 18\times\frac{4}{3} = 24$.

The above process of reasoning can be accomplished with illustrations. And also, we can make students practically divide physical objects to understand this concept. This is the first stage towards learning how to divide by a fraction.

In this case, students must be asked to generalize the rule of fraction division “a number divided by a fraction equals to the number multiplied by the reciprocal of this fraction”, that is $N \div \frac{b}{a} = N \times \frac{a}{b}$.

And how to “make students perfect” with this rule, i.e. to get familiar with it and remember it? They need to perform proper exercises. First they should do some similar fraction divisions, such as $15\div\frac{3}{4}$, $20\div\frac{5}{8}$, $28\div\frac{4}{5}$ etc., and then they need some contrastive exercises to compare the fractional multiplication and division, such as $18\times\frac{2}{3}$, $18\div\frac{2}{3}$ etc. and some composite exercise, such as $\frac{4}{5}\div\frac{2}{9}$, $\frac{2}{5}\times\frac{3}{4}$, $\frac{2}{9}\div\frac{1}{3}$, $\frac{2}{7}\times\frac{1}{6}$, $\frac{2}{3}\div5$ etc..

Only in this way can the basic knowledge and skills necessary in performing fraction division be solidified. This is what we call memorization based on understanding and "practice make perfect".

Consolidation and accumulation of knowledge

According to psychological research, memorizing and preserving information are important parts of studying. Only after the student has memorized and mastered the core concepts of mathematics, the basic skills and the basic methods, can he or she freely apply that knowledge to the real world. It is necessary for students to consolidate those frequently used mathematics contents, and those contents will be needed in their further study also. Some important mathematics contents always need to be memorized, and it must be repeated if it is to be memorized.

For example, the basic multiplication table (from1×1 to 9×9) has to be memorized, the Pythagorean theorem has to be remembered, and many other crucial computational methods and problem solving strategies have to be well known. This is not only for students' further study, but also for their future work and personal lives as well.

To support memorizing and preserving important information, we designed some exercises to help students' understanding, instead of simple memorization.

For example, the trigonometric formula $\sin^2\alpha + \cos^2\alpha = 1$ is an important formula. At the beginning of instruction of trigonometry, when we introduced the values of sine and cosine of special angles, we designed this kind of exercise:

To calculate the following values and find out what is special about them.

（1）$\sin^2 30^\circ + \cos^2 30^\circ$, $\sin^2 60^\circ + \cos^2 60^\circ$, $\sin^2 45^\circ + \cos^2 45^\circ$ ；

（2）$\sin 30^\circ - \cos 60^\circ$, $\sin 60^\circ - \cos 30^\circ$, $\sin 45^\circ - \cos 45^\circ$ ；

（3）$\sin^2 30^\circ + \sin^2 60^\circ$, $\cos^2 30^\circ + \cos^2 60^\circ$, $\sin^2 45^\circ + \sin^2 45^\circ$.

First, this exercise let students memorize the values of sine and cosine of special angles, and then encourage them to find out what is special in the outcomes. It is very beneficial to learning the formulas $\sin^2\alpha + \cos^2\alpha = 1$ and $\sin(90°-\alpha) = \cos\alpha$ at a later time. In mathematics learning, it is important for us to explore, to investigate, and to gain knowledge actively, and it is also important for us to consolidate and store knowledge.

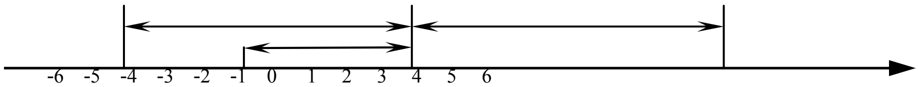
Chinese math education has a good tradition, which is of focusing on building a solid foundation. This foundation emphasizes basic knowledge and basic skills. In order to master the basic knowledge and skills (to freely use them in further study or real world problem solving) necessary to comprehend the essence of mathematics, a certain amount of practice is needed to consolidate a student's knowledge.

In some sense, studying means accumulating. Practising enables students to accumulate mathematics knowledge.

Whenever a new concept is introduced, students are first exposed to basic exercises, so that the knowledge will be reviewed and solidified. Once the basic exercises have laid the foundation, variant exercises are provided, which help students to look at the new concept from different directions, to relate it to the existing knowledge in their minds, and practice with problems of increasing depth. These varied exercises complete the mathematical study by training students to apply the acquired knowledge in a comprehensive step-by-step manner.

For *example*, the absolute value of a rational number is an important concept. Many students experience difficulty with it in the beginning. We can guide students' understanding of this concept gradually with well-developed and organized exercises.

First we use the number line to illustrate things. Every rational number corresponds to a point on the number line, and the distance between this point and the origin is the absolute value of this number.



According to this illustration:

$|5| = 5$, $|-5| = 5$, $|-3| = 3$, $|0| = 0$,

Next, we ask students to do some basic exercises, such as:

Write down the absolute value of the following numbers.

6 , -8 , -3.9, $\frac{5}{2}$, $-\frac{2}{11}$, 79, 0, -15.6

Then, we present them with variant exercises, such as:

1) Compare the value of each following pair of numbers.

   -2.5 and $|-2.5|$, $|-0.6|$ and 0.6 , $|-0.5|$ and $\left|\frac{1}{2}\right|$.

2) Fill in the blanks.

   If $a > 0$, $|a| =$ _________ ,

   If $a < 0$, $|a| =$ _________ ,

   If $a = 0$, $|a| =$ _________。

3) Multiple choices.

   If $a < 5$, $|a - 5| =$ ______.

   (A) $a-5$, (B) $5-a$, (C) $\pm(a-5)$

In this way, we start from what the absolute value of rational numbers is, to move on to how to find the absolute value of a specific rational number, and finally to what the absolute value of a quantity represented with a letter or an expression is. We push students' thinking step by step towards a deep comprehensive understanding.

Formalization and normalization of knowledge

Working on exercises is a process of refinement. Information is different from knowledge. A student has to refine the information he or she has learned from various sources into useful knowledge. Unorganized information is not knowledge. It cannot be used to solve problems.

It is important to protect students' special aptitudes and their originality, but proper formalization and normalization are necessary in mathematics learning. We need to strike a balance between encouraging student's originality and normalizing what they have done. That is to say, that although students can use different ways to investigate mathematics problems, they still need to understand and master the mathematics that the majority of the world understands. To be more specific, we encourage students to take the initiative to help them develop the habit of critical thinking, rather than allowing themselves to be confused by unorganized thoughts.

Take for example, the diversity of computational methods, which is being advocated in China lately. When primary school students are learning the four fundamental operations of whole numbers, they can usually develop their own algorithms within the proper contextual learning activities. We advocate diversity (*encouraging students to think individually*). The goal is to encourage student's thinking actively, rather than to let them think aimlessly. The goal of diversity is to help students understand the meaning and the rationality of calculation methods. We emphasize the training of thinking without advocating the senseless diversity of methods. That is to say, the ultimate goal is not to have a great quantity of methods, but much high quality training that facilitates students' thinking.

About this issue, we have been asked, "After we have promoted a variety of exercises among students, do we still need to lead them to optimize their thought?" The answer can be summed up in two points: First, after presenting the various exercises, we do need to lead students to optimize their understanding. We should direct them to think about what methods are the most appropriate and most convenient ones. Second, students should accomplish optimization by themselves. It is a process of students' rethinking and understanding. Our teaching should stimulate students to analyze their own experiments, and to communicate about their findings with their classmates, to help them understand what scientific and normalized methods are easy to operate.

For example, when primary students learn addition and subtraction up to 20, they are presented with the problem: 13 – 5 = ? Students propose:

- To calculate 13 – 5 , we can follow the path 13 – 3 = 10 , 10 – 2 = 8 ;
- Or, the path 10 – 5 = 5 , 5 + 3 = 8 ;
- Also, the path 8 + 5 = 13 , so 13 – 5 = 8

In this situation, as long as the students' thoughts are reasonable, the teacher should always be approving, promoting students to gain a further understanding at a later time. Students' practising is very important here. Firstly, practising enables students to understand the meaning of subtraction and learn calculation methods. Also, in their exercises, students would gradually discover the quickest way of calculating and draw conclusions, optimizing their methods.

In some exercises, we train students how to think by themselves. For instance, in order to introduce common algorithms and have students form their own knowledge, we designed exercises like:

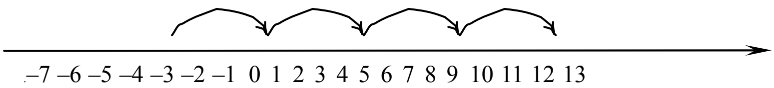
8+□=13 , □+5=13 , 9+□=16 , □+9=16, etc.

Questions designed in this way will help students consider the possibility of using addition when doing subtraction, "do subtraction by addition", thus helping students develop the habit of using this calculation method in the future. This type of calculation is the basic of the four fundamental operations of arithmetic, and they would be used frequently. So, we believe the formation and familiarization of this fundamental and effective skill will greatly benefit students' entire lives.

Rational number calculations can be taken as another example concerning normalization.

While attempting to understand the multiplication of rational numbers, a student comes up with the equation (-3) x (-4) = 9. The student explains his logic as follows:

Starting from the position of -3 on the number line, he moves in the positive direction 4 times in increments of 3, and arrives at 9.



Although this idea is very creative and original, it is still incorrect.

In the mathematics teaching process, it is necessary for students to take the initiative and form their own understanding of the material, but it is also important for the educator to provide proper guidance. Yet, how does an educator lead the student to the correct conclusion? If we need to guide this student to normalize his thought, how do we do that? This question may be left for everybody to find out.

I think, in order to solve such kinds of problems, it is necessary to direct students to perform exercises with certain conditions, to guide students' thinking to correct conclusions. Here the guidance from the teacher is necessary and important.

How to guide students' practising effectively

About how to guide students, I would like to explain two points. One is enlightened teaching, the other is variant teaching.

In Chinese mathematics instruction, we frequently use enlightened teaching and variant teaching strategies. I think they are effective strategies to stimulate students to think actively, and to help them think deeply, in order for them to grasp the essence of mathematics. So, we designed many activities of practising using these strategies, whether in the classroom teaching or in the exercises in textbooks.

Enlightened teaching

One thing that must be emphasized here is that, although we can use questions or exercises to help students with their understanding, thinking, consolidating and normalizing what they have learnt, it is not the same as the process of mechanical repetition, memorization or simple imitation.

Enlightened teaching is an excellent tradition in Chinese instruction. In the teaching process, we should provide students with enlightening questions or exercises that will stimulate students' critical thinking. To provide well-designed questions at the right time in the right place can make students good at understanding.

For example, in the teaching of trigonometry how can a question be designed to enlighten students in deriving these translation formulae?

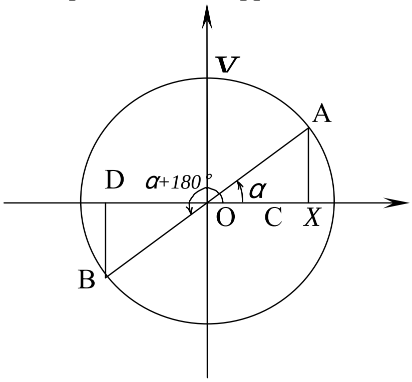
$\sin(180^\circ - \alpha) = \sin\alpha$, $\sin(180^\circ + \alpha) = -\sin\alpha$, $\sin(360^\circ - \alpha) = -\sin\alpha$, $\sin(360^\circ + \alpha) = \sin\alpha$; $\cos(180^\circ - \alpha) = -\cos\alpha$, $\cos(180^\circ + \alpha) = -\cos\alpha$, $\cos(360^\circ - \alpha) = \cos\alpha$, $\cos(360^\circ + \alpha) = \cos\alpha$.

Can we set questions like these?

Question 1: Can you derive the trigonometric translation formulae from the geometric properties of a circle?

This type of questions is too broad (there are so many geometric properties in a circle). Students have no idea what a trigonometric translation formula is at this time; hence they could not possibly understand what the question is asking.

Question 2: Look at this diagram. Is there any relationship between the opposite side of α and the ending side of α+180° in the unit circle? Can you derive the relationship between sin α and sin (α+180°)? This question is so specific that there is little space for the student to discover the relationships on his or her own.

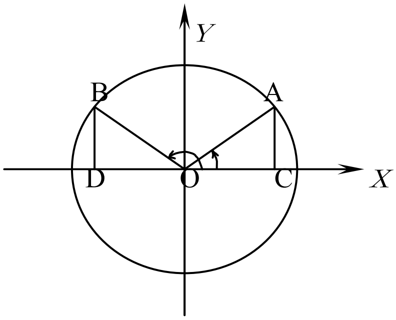


Question 3: We can find the values of the trigonometric functions of acute angles in a table, but how do we find the values of the trigonometric function of arbitrary angles? Can we convert the trigonometric functions of arbitrary angles to acute angles?

This question is too "utilitarion", without any connotations, and barely related to the meaning of deriving the formula. Hence it cannot help students to explore the translation formulae.

Therefore, we designed the following conditions in our textbook:

First, it is made clear that the trigonometric functions are closely related to the unit circle. And the basic relationship in the trigonometric functions can be illustrated using line segments inside of unit circle.



The circle is perfectly symmetrical along any diameter. Suppose ∠AOC and ∠BOC have the same starting side on the positive direction of the X-axis, but the "opposite" side of∠BOC is symmetrical with the "opposite"side of ∠AOC along the Y-axis. After presenting this diagram, we then ask:

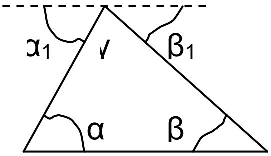
Can you use the unit circle and its symmetry properties to discuss the relationship between ∠AOC and ∠BOC, as well as the relationship among the trigonometric functions of the two angles?

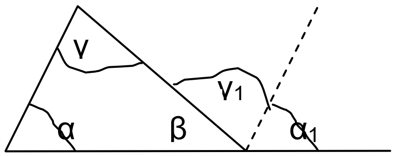
Questions like this link new knowledge with old. By this appropriate question students can be stimulated to think and explore the relationships, and to discover the translation formulae of trigonometric functions actively.

Enlightening questions are not only useful in teaching new concepts, but also are beneficial to students' further study.

For example: When students are learning the property of triangles and that the sum of the three inner angles will always be 180°, we can start with putting together shapes. On the basis of conjoining, students can use the mathematics knowledge they learned to prove statements in the theory.

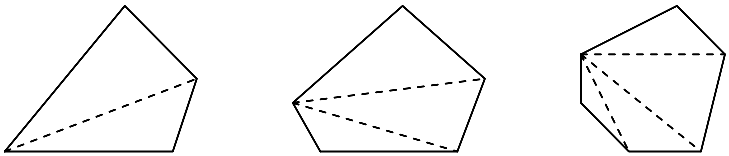
In real-world teaching experience, students usually come up with the two results below:

(1) 

(2) 

Relating to the conjunction shown in illustration (2), the teacher can go ahead and stimulate students to discover the relationship between the outer angles and the inner angles of a triangle. In addition, we have designed other exercises, as presented below, in problems for practising following the introduction of this concept.

Ask students to draw all the diagonals of each of the polygons below, and then calculate the sum of the inner angles of each polygon.



When students have finished drawing the diagonals of these polygons, they can soon relate the knowledge of the sum of inner angles of a triangle to calculate the sum of inner angles of each polygon.

However, this is not the end of the story. Our goal is to help students deduce more general conclusion, namely the relationship between the number of sides of a polygon and the sum of its inner angles. We recommend leaving this problem for students to investigate individually, and then come back together to discuss with other students and summarize their findings. Finally, students get the result: in an N-polygon the sum of the inner angles is $180° \times (N - 2)$. This inspiring and enlightening teaching method let students construct mathematics within an efficient environment. We encourage students to research actively, and in the mean time we prevent their incorrect reasoning, so they can stay on the right track.

Variant teaching

It is very important for students to continually analyze and revisit the material when they have learned some new content, but repeating similar questions or exercises senselessly is a meaningless job. Chinese mathematics teachers usually lead students doing practice by using a variant teaching strategy. If we consider that the enlightened teaching is a good way to introduce students into the field of first getting to know mathematics, then variant teaching is a way to guide students' deeper thinking, to introduce them to getting greater understanding of mathematics.

We noticed that the Swedish educational expert F. Marton's theory is correlated with our variant teaching. He called his theory "phenomenograph", the kernel of which is distinction and variation. In his opinion, studying means distinguishing, i.e. paying attention to the essence of the researched objects. (See *Yuxin Zheng* "Paradox theory of Chinese learners", *Mathematics Education Journal*, *2001, vol.2*)

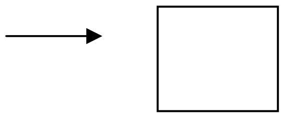
I would also like to mention a Chinese author, Dr. Lingyuan Gu, the Director of Shanghai Education Science Research Institute, who developed a theory "variation in teaching", that explains how variant teaching gets from conceptual variation to process variation. (See *Lingyuan Gu* "Variation teaching research", *Mathematics Education*, *2003, vol. 1*)

The idea of "conceptual variation" is to look at a concept from different perspectives by using some contrasting and counter examples to recognize its essence.

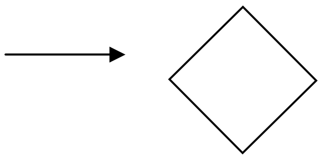
When introducing the characteristics of shapes, we can design a few exercises on comparison and recognition. Through comparison, students can gain a better understanding of the characteristics of various shapes.

For example, after students have been introduced to the idea of perpendicular lines, we ask them: "Are the lines perpendicular to each other in these two figures?"

Normally in children's mind a square is like this,



then we turn it and ask: "It is a square too, is not it?"



When teaching the concepts of angle and parallelogram, we let students recognize them: "Is angle A larger than angle B？"

"Are both figures parallelograms?"

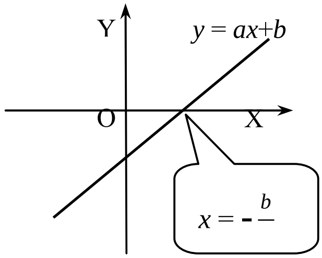
Such kinds of variant exercises enable students to think more carefully, to help them understand and draw conclusions from various figures, thus acquire a better comprehension.

"Procedural variation" is to consider the various possible applications of related mathematic concepts, to get further understanding of the learned materials, and to stimulate students' grasp of the essentials of mathematical concepts and the assimilation of a new knowledge into their existing knowledge systems, thus constructing their own network of mathematical knowledge in their minds.

In our textbooks, we put together a variety of contextual questions to make students compare and contrast the reasoning behind each problem.

For example, before students study linear functions, they have learned how to solve a linear equation with one variable, a linear inequality with one variable, and the set of linear equations with two variables.

After teaching the concept of linear function, we designed a section of "studying equations, inequalities and the set of equations from the perspective of function".



Observe the illustration of a linear function, and think: *If the value of function* **y = ax+b** (*a* $\neq 0$) *is 0, then x = ?*

This is the solution of the equation ***ax+b* = 0** (*a* ≠ *0*): $x=-\frac{b}{a}$.

Observe the illustration of a linear function, and think: When the value of function **y = ax+b** (a ≠ 0) is greater than 0 (or less than 0) then x= ?

This is the solution of the collection of inequalities ***ax+b* > 0** (*a* > *0*): $x>-\frac{b}{a}$,

and the solution of the collection of inequalities ***ax+b* < 0** (*a* > 0): $x<-\frac{b}{a}$.

Compare and contrast a linear function and the set of linear equations with two variables, think : What do their illustrations look like? How can we illustrate the solution of a linear equation with two variables?

Because the two-variable linear equation ***ax* + *by* = *c*** can be converted into the form of a linear function $y=\frac{-a}{b}x+c \quad (b\neq 0)$, so the illustration of this linear equation with two variables is a straight line. Then, to find out the solution of a system of linear equations with two variables,

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$$

we can look for the point of intersection formed by the two straight lines. The coordinate of the intersection point is the solution of a pair of linear equations.

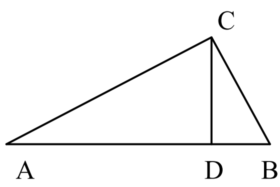
Through this kind of practice, students would get further understanding of linear functions, linear equations (and inequalities) with one variable, and systemsof linear equations with two variables.

For another example, after having taught similar triangles, we ask students to prove the statement below.

Knowing that in Rt△ABC，∠C=90°

CD is the height of the hypotenuse，

Prove: △ABC∽△CBD∽△ACD。



This problem is easy to solve. If we expose the students to varied exercises, they can improve their achievement.

We ask students to think: According to the properties of similar triangles, what is the relationship among their corresponding sides?

As a result, students deduced $AC^2 = AB \cdot AD$, $BC^2 = AB \cdot BD$ and $CD^2 = AD \cdot BD$, which is a very useful theorem - the projection theorem. If we encourage students to keep exploring and deducing, they will eventually get:

$$AC^2 + BC^2 = AB \cdot AD + AB \cdot BD = AB^2$$

And this is the famous Pythagorean theorem!

Students can see the relationships amongst the related contents. How beneficial the result can be if we stimulate students to think critically and perform variant exercises!

Conclusion

In my presentation above, I stressed the importance and necessity of practising (including doing exercises) in mathematics learning. This does not mean studying mathematics only by doing exercise, but to try to find a balance between the teacher's teaching and students' learning, between the students' investigational activities and exercise work.

That is to say, although we encourage students to learn actively, the teacher's guidance is still important in students' mathematics learning. It also includes the effects of textbook design. The design of examples and exercises in textbooks reflects the ideas of educators. By engaging in well-prepared reasonable practising, students can enhance the power of their thinking, deepen their understanding of what they learned, and master and apply mathematics successfully.

In conclusion, practising is still necessary for mathematics learning. Practice should be appropriate, i.e. well selected and well arranged. Enlightened teaching and variant teaching are good experiences in Chinese mathematics teaching.