The "Two Basics" mathematics teaching approach and open ended problem solving in China

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Abstract

There is a tradition of advocating the "two basics" – basic knowledge and basic skills – in Chinese mathematics education. The direct consequence is that Chinese students are able to display excellent performance in the International Mathematics Examinations (e. g., IAEP 1989), and outstanding results in the International Mathematics Olympiads competitions. In this article, we will present why and how Chinese teachers teach the "two basics", and discuss how to combine the pupil's creativity with these "two basics". Open ended problem solving is a way to meet the goal. The following topics will be considered: Cultural background; speed of computation; "practice makes perfect"; efficiency in the classroom; the balance between "two basics" and personal development. In particular, Chinese mathematics educators pay increasing attention to the link between open ended problem solving and the "two basics" principle.

Introduction

Due to historical reasons, in many Eastern countries, including Japan, Korea, Singapore, and China (Mainland China, Taiwan, and Hong Kong), and even Russia, mathematics educators emphasize the importance of foundational training more than is ususally seen in the West. The principle of the "two basics" (basic knowledge and basic skills), however, is most typically observed in Mainland China.

A direct consequence of the implementation of the "two basics" principle in Mainland China is: acquiring a leading position in numerous international mathematics assessment programmes and contests, for example, topping the accuracy rate list in the IAEP 1989 (International Assessment of Education Progress, 1989), and achieving outstanding results in the past International Mathematics Olympiads.

There is a paradox, however. On one hand, Chinese students are successful in many international mathematics tests. On the other hand, mathematics teaching and learning in China seems to be oriented towards rote memorization and repetitive exercises. In our opinion, the notion of "two basics" teaching approaches may be the key to resolving this paradox.

In this paper, we will illuminate the meaning of the "two basics" mathematics teaching approach, and the link between this principle and "open ended problem solving" in Mainland China.

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The four dimensions of the "two basics" principle

In Mainland China, the "two basics" principle in mathematics teaching is a broad and loose concept without a strict definition. Its general meaning is that in the aspects of "solid foundation" and "individual development and creativity", although both are important foundation is the more important. Most Chinese mathematicians and mathematics educators believe that "although a tower is beautiful its groundwork is more important."

In fact, primary and secondary education is foundational education. Establishing a good mathematics foundation is the main task of fundamental education. Especially, this foundation can only be laid properly when one is young, or else it would be too late. We lay the foundations in our childhood's mathematics practice, just like language and piano-playing.

Therefore it is of foremost importance that a good foundation is laid during school years. Without a solid foundation, it is impossible to realize creativity, let alone students' ultimate differentiated individual development.

The educational idea of the "two basics" mathematics teaching approach can be shown in the following four dimensions: Calculation speed: speed leads to efficiency; Procedure memorization: understanding through memorization; Accuracy in expression: based on logical analysis; Doing exercises: repetition with variation.

Calculation speed: leads to efficiency

Calculatoin speed is an important factor of the "two basics" mathematics teaching approach. As is well known, calculation skill is a procedure and mathematical thinking is a process. In order to enhance the efficiency of mathematical thinking, we must keep an adequate speed of calculation. In fact, speed will save working memory space for thinking at a higher level, i.e., speed leads to efficiency. In Chinese primary schools, fast and accurate calculation with the four operations involving integers, decimals and fractions is an essential requirement. Let us refer to the Fenghua Survey[3].

Test contents: First graders (7 years old): A total of 90 sums have to be completed within 15 minutes.

(1) 50 questions on addition and subtraction under 100

3+13=	16+4=	50+5=	12+60=	2+57=
60+33=	53+9=	28+10=	30+48=	7+12=
9+17=	40+45=	23+8=	12+34=	86+8=
78–30=	20–12=	15–5=	20–15=	81–80=
100–40=	95–70=	27–4=	40–20=	18–12=
16–9=	36–16=	26–11=	47–8=	97–14=
45+1=	96–4=	63+7=	16–2=	37+6=
99–2=	38+50=	76–9=	8+25=	63–6=
80–40+30=	17+8+30=	49+20–60=	44–10+13=	35–20+15=
18–0+9=	10+60–8=	32+9–20=	98–30+11=	20–8–11=

(2) Fill in the 40 blanks:

2 + 5 = (   ) – 5 = (   ) + 7 = (   ) + 2 = (   )
11 – 2 = (   ) + 9 = (   ) – 7 = (   ) + 6 = (   )
15 – 8 = (   ) + 2 = (   ) – 3 = (   ) + 18 = (   )

[3] It was conducted by Zhou Leiming and Hu Yixiang. 2002.

85 – 40 = ( ) – 20 = ( ) – 5 = ( ) – 11 = ( )
12 + 7 = ( ) + 9 = ( ) – 17 = ( ) + 16 = ( )
19 + 7 = ( ) – 8 = ( ) + 13 = ( ) + 7 = ( )
76 – 20 = ( ) + 14 = ( ) – 30 = ( ) + 9 = ( )
95 – 61 = ( ) + 18 = ( ) – 12 = ( ) +31 = ( )
31 + 16 = ( ) – 15 = ( ) + 42 = ( ) – 50 = ( )
41 – 20 = ( ) + 19 = ( ) – 14 = ( ) + 5 = ( )

School	Sample size	Quickest	Slowest	Average time used	Average grade	Excellence ratio	Passing rate
Jinpeng town centre school	48	6min	15 min	9 min 38sec	90.1	46.65%	100%
Xi'qi School	23	6 min 3sec	14 min	9 min 52sec	87.4	36.7%	96%

Table 1.Results and analysis of the mathematical ability of first graders

Conclusion: Chinese primary students can complete 10 test questions of addition and subtraction with numbers under 100 in one minute. Is it necessary? We are not sure. However, this is an ordinary speed in China.

We will show two other results on "calculation speed". Third graders (9 years old), can complete 2 test questions per minute on average. Some of the questions are:

125×72=	8500÷50=	4907÷7=	490×20=
8640÷60=	8585÷17=	2821×3=	490÷35=

In a survey with a sample size of 6000, we hope to measure the speed of algebraic manipulation on polynomials. Students are asked to complete 38 test questions in 10 minutes (cf. the Appendix). Some of the test questions are:

Solve questions 1 to 5:

1) $-(2/3)\, ab + (3/4)\, ab + ab =$
2) $-y^2 - 2x^2 - (-3y^2) =$
3) $3x^2 y \cdot (1/2)\, x \cdot (-2xy^2) =$
4) $3x^2 y + (1/3)\, xy \div (-xy) =$
5) $[(-2n)^2]^3 =$

Factorization (1–3):

1) $(a + b)^2 - (x - y)^2$
2) $(x + y)^2 + 5\,(x + y) + 6$
3) $(m - n)^2 + 4(m - n) + 4$

Completing the square:

$(1/2)\, x^2 + x + (2/3)$

The conclusion are summarised in the following table displaying the average number of correct questions per student, the standard deviation and the accuracy

Grade / Variable	8th	9th	10th	11th	12th	Total
X	17.82	20.73	28.11	29.43	31.00	25.78
σ_n	6.08	7.05	4.22	4.34	4.20	7.29
$Y(\%)$	46.89	54.55	73.97	77.45	81.58	67.84

We think this may be too demanding for school students. However, speed requirements are still a goal of mathematics teaching and learning in China. "Quick calculation competitions" are held in most schools year after year. In fact, calculation speed is the basis for higher achievement in any time-limited test.

Procedure memorization: understanding through memorization

Nobody denies the importance of memorization in processes of cognition. However, there are different views concerning the relationship between memorization and understanding. For example, many educators strongly demand that "Don't learn anything by rote!" while others say that "understanding happens through memorization!"

Most Chinese teachers believe in "first memorize it, and then understand it step by step." For example, although children do not understand why they should undertake piano finger exercises, they have to memorize them, and then understand things later. Similarly, our ability to speak our mother tongue just relies on memorization and imitation, even if we do not understand what the grammar is. In China, we usually say: "if you want to understand something, you should practice it; even if you do not understand it well, you have to practice, too. In the process of doing you will understand things better and better."

Here are some examples.

(1) All pupils must memorize and recite the 9×9 multiplication table when they are 8–9 years old (in the second-third grade).

(2) The rule "negative times negative equal to positive" ("–"×"–"="+"). To understand this algorithm is more difficult. Up to today, we are still unable to explain why it is correct in a convincing manner. In China, most pupils memorize the rule first, and then understand it gradually.

(3) For the trigonometric formulae, in actual teaching, students are required to recite angle-sum formulae, double-angle formulae and half-angle formulae and the formulae for changing a trigonometric sum to a product and vice versa, as far as Sine, Cosine and Tangent are concerned. (Formulae for triple-angles have not been required for students to memorize in recent years). This is basic knowledge for students' life-long study, especially in the study of calculus.

These views are also shared by some Western scholars. For instance, "memorizing is understanding; understanding through memorization" (Marton, 1991). Memorization is a means of deepening understanding or is a precondition for understanding? An interpretation derived from studies of Japanese students is that repetition may be a route to understanding (Hess and Azumas, 1991).

Accuracy in expression: based on logical analysis

In China, mathematics teaching is required to explain mathematical concepts and mathematical ideas through informal approaches. However, to some extent it is necessary to keep up the mathematics with accuracy, logic and formality. In particular, the mathematical language should be formal. Students have to exercise a lot to learn how to make logical expressions.

An example is that mathematics teaching in China stresses rigorous deduction and proof. Between "verification" and "proof" the latter is the more valued. Let us take a basic piece of geometric knowledge – the Gou Gu Theorem (Pythagoras' Theorem) - as an example. In teaching this theorem must be proved in an abstract manner using algebraic or geometric methods, while cut-and-paste method is not counted as an acceptable way for a definitive proof.

Here is another example that appeared in the 1991 National College Entrance Examination:

(Question 15) Which of the following statements is false?

(A) There exists values of α and β such that $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$,

(B) There does not exist infinitely many values of α and β, such that $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$,

(C) For any α and β $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

(D) There exists no values of α and β such that $\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$,

The correct choice is (A). The solution process should conform to rigorous logical reasoning providing sufficient reason and being expressed in a clear and formal way.

The "two basics" mathematics teaching approach insists on keeping mathematical thinking in an abstract style. According to a report by Jinfa Cai (University of Delaware, USA), in contrast with pupils from USA, Chinese pupils take a different way to solve the fraction problems. The test question is:

The Pizza Ratio Problem: 8 girls share two pizzas and 3 boys share one pizza, all pizzas have the same size. Please justify whether each girl or each boy gets more pizza.

There are (at least) three ways to solve this problem:

☐In numerical symbols: $2\div 8=1/4$. $1\div 3=1/3$. $1/3 > 1/4$.

☐In words: 2/8 equal to 1/4, and 1/3 is greater than 1/4, so a boy gets more pizza.

☐By drawings

Small pieces Large pieces

Cai's report shows that American pupils prefer to use the drawing way to solve the problem, while very few Chinese pupils handle it in this way.

	China	China	China	USA	USA	USA
Grade	4	5	6	4	5	6
Correct rate	21	57	93	42	53	59
Visual drawing	4	3	4	65	53	59
Numerical symbol	35	51	58	27	36	47
Written words	61	42	28	8	11	4

As indicated above, the "two basics" approach seems to pay more attention to logical analysis, abstract thinking and formal expression than do some Western approaches.

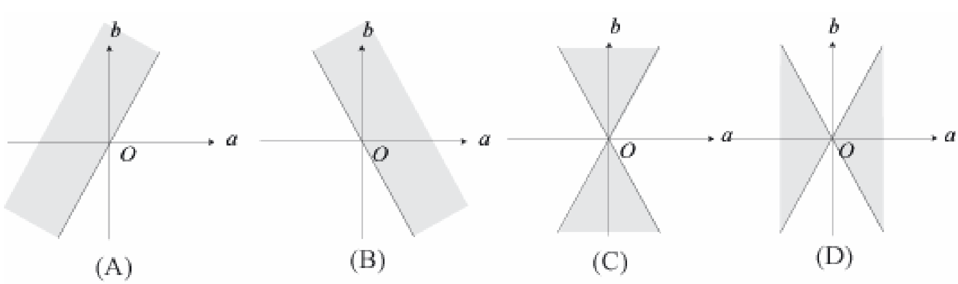
Doing exercises: repetition with variation

The attaining of a mathematical skill must take place through frequent practice. Of course, doing mechanical repetitive exercises does not suffice to achieve individual development. Recently, lots of research studies report that repetition with variation is a Chinese way of promoting effective mathematics learning (Gu, Huang , Marton).

Let us consider a typical example of variation (only one letter change: $c \rightarrow a$)

Example. (A test question of the College Entrance Examination in 2003)

If the graph of the function $y = ax^2+bx+a$ has two intersection points with the x-axis, then the point (a, b) located in the area of ( ).



The correct answer is (C), but $a \neq 0$.

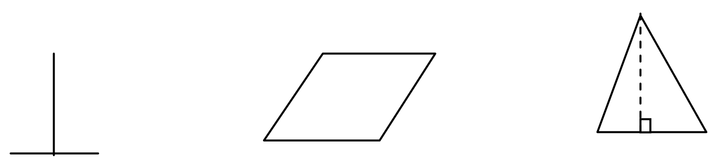
Although we changed only one letter in the normal expression of $y = ax^2+bx+c$, the function $y = ax^2 + bx+a$ will concern a lot of basic mathematical knowledge:

- The roots of the quadratic equation are real;
- The discriminant is: $b^2 – 4aa = b^2 – 4 a^2 = (b + 2a) (b – 2a) \geq 0$;
- Equation of a straight line;
- Areas bounded by lines
- The exceptional straight line $a = 0$.

The major form is the "variation method", including logical variation of concepts and the variation in the process of solution. There are explicit variations as well as implicit variations. This sophisticated arrangement of exercise is much more effective than the simple repetition of tasks (Huang, 2002). Many studies show that continuous practice with increasing variation will lead to understanding (Marton, 1997).

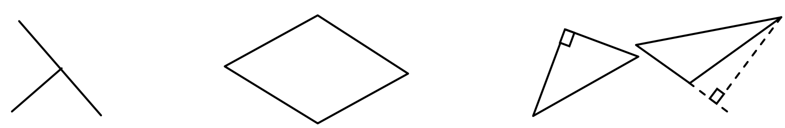
There are two types of variations:

(1) Conceptual Variation (I)

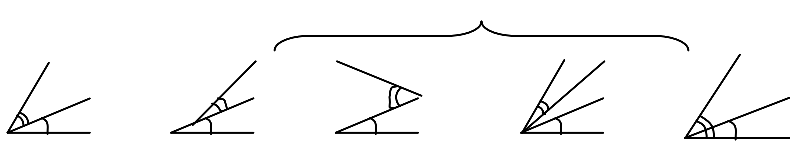


Perpendicular

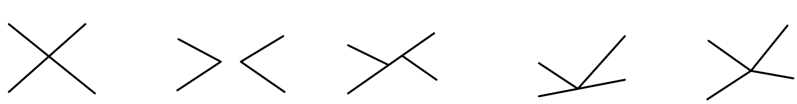
Conceptual variation (II)



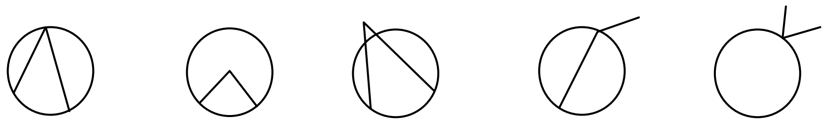
Adjacent angle



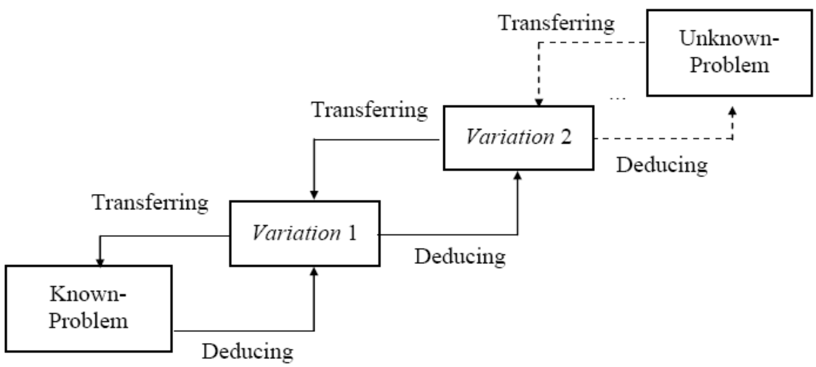
Opposite angle



Angle at the circumference



(2) Process variation



Characteristics of mathematics teaching under the “two basics” principle

China is a developing country with a population of 1.3 billion people. There is a nine years compulsory education system. As the class size can reach as many as 40–50 students, it is in practice impossible for teachers to practice individualized teaching. Under the influence of the “two basics” principle, we can consider some characteristics of mathematics teaching in China.

The rhythm of classroom teaching is led by the teacher’s plan

It emphasizes inspiring teaching and opposes spoon-feeding teaching. Students are required to adapt to the pace of progress set for most students by the teacher. It requires teachers to present the main mathematics contents as quickly as possible so as to avoid students spending too much time on winding paths. However, in Chinese classrooms, teachers do not always give lecture demonstrations. Oral questioning is quite frequent. Usually, the teacher presents a series of relative easy questions and asks students to answer in groups or as individuals, thus leading them to reach the set target through a number of small steps instead of letting them discover things on their own. This is what we called “small steps” teaching.

The use of the “essential lectures, much practice” principle

Regarding the relationship between comprehension and manipulation, this principle does not support the proposal of “understanding first” but maintains that both are equally important. The lecture demonstration for understanding must be short and focus on essentials in order to leave more time for solving mathematical problems. As to time allocation, there is no need to spend too much time on the comprehension stage, as it is not likely to be accomplished at the first explanation. There must be practice in mathematics. Therefore, doing exercises in mathematics is of course necessary following comprehension, but even without thorough understanding, students can still do exercises, and they can develop their understanding through working on the problems.

Emphasizing logical expression
The "two basics" principle emphasizes the importance of fostering mathematical thinking, and the effect of repeatedly explaining and training in the learning of mathematical subject matter and ideas, such as complete categorization, conversion among the four propositions, and comprehension of necessary and sufficient conditions; analysis, induction, synthesis, association, and RMI methods etc. In particular, during pre-examination revision periods, the emphasis is on learning new things through reviewing the old. Some outstanding teachers use teaching methods that are intensive, fast-paced, and full of content, which can help them go over some twenty or thirty problems in a single mathematics revision lesson, and then generalize the mathematical methods employed. This kind of skill training is a demonstration of applying the "two basics" principle on its highest level.

Pay more attentionsto the application of "mathematics method"
Through research on "thinking methods in secondary school mathematics"(e.g., synthesis and analysis, inductione and deduction, transformatoin between number and shape, modeling, analogy, connection etc), mechanical logical reasoning evolves to a logical thinking ability, so that one can grasp the overall structure of mathematical thinking at the secondary level, and develop systematic knowledge about it. It has been revealed that well versed and flexible mathematical manipulation can facilitate the formation of mathematical concepts. The ability to apply formulae and patterns in calculation can convert mechanical manipulation to real mathematical calculation ability. Skillful calculation and memorization of formulae can make mathematical thinking more condensed and faster, leading to a progression to a higher level of mathematical thinking (Hess and Azuma, 1991; Thomas and Bain, 1984; Li, 1996).

Historical roots and social environment for the "two basics" principle in mathematics teaching

A good foundation is essential for the construction of a building. So, of course no one would deny the importance of a good foundation, but the question is the degree or level to which it should be should emphasized. People have different views and practices concerning this question. Most Chinese educators' beliefs were gradually formed under the influences of culture accumulated throughout thousands of years.

Let us trace back to factors forming these beliefs about foundation in China's educational tradition.

First, thousands of years of agricultural culture, especially the culture developed from plantation of paddy-field crops, required detailed and crafty work. Given a small land area, farmers had to rely on well-practiced and efficient techniques to obtain maximum outputs. This is very different from a nomadic society's culture where people can make a living through the extension of rearing areas. Thus, in the Chinese society, to be quipped with effective and efficient "skills" are of vital importance for survival.

Secondly, the strict examination system and unified exam questions have driven students to only learn the contents that will be tested in the exams. The system of civil examinations in China can be traced back to as early as the year of 597. Through this system, peasants can become government officials if they can pass the national examinations. This is a highly fair and civilized policy. It is hence rooted in the minds of Chinese that examinations can determine one's life. After the Ming dynasty, the test items used in the unified examinations by the government became "Bagu-oriented" (which refers to a set of extremely condensed and procedure-fixed basic knowledge, and very stereotyped and sophisticated writing skills). Referring to the modern mathematics examinations, most of the contents tested are also mathematical procedures and well-practiced skills. As examinations focused on the "basics", for the purpose of scoring well on the examinations students would also learn the basics only (Bishop, 1998; Zhang and Lee, 1990).

Thirdly, as educational wisdom in the Chinese society has it, "practice makes perfect". It was always believed that the primary aim of education is to achieve familiarity. Through familiarity, one can naturally become "skillful" (perfect).

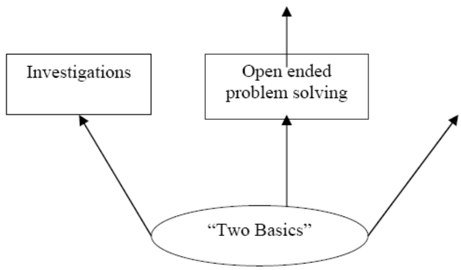
Finally, in the 1950s, the mathematics education in Mainland China was heavily influenced by mathematics education in the Soviet Union. As is well known, mathematics education in the Soviet Union in 1950s emphasized the memorization of rules and regulations of basic knowledge, and the rigour of proof, including the basic training of logical reasoning.

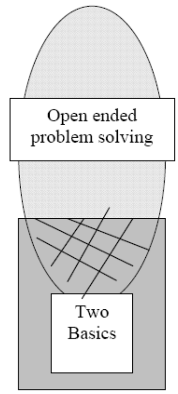
Under the above factors, in 1963, the Ministry of Education of China stated the following in the mathematics syllabus: "mathematics education should strengthen students' learning of basic knowledge and basic skills". Moreover mathematics instruction should foster students' "basic computational ability, spatial imagination ability and logical thinking ability". This idea of placing emphasis on the basics is still in force today.

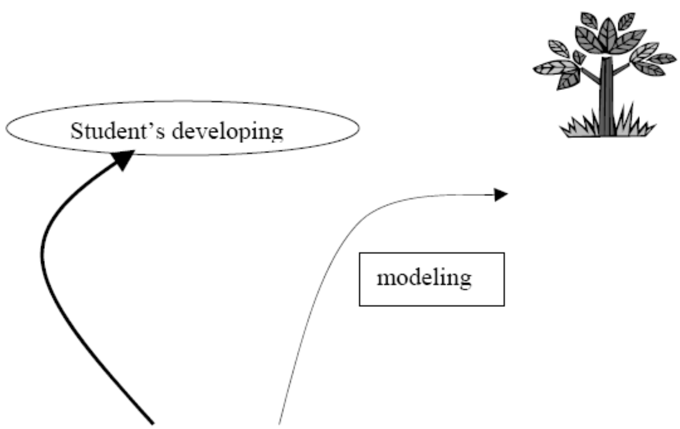
An ongoing development: Open ended problem solving

We do not get a complete picture if we only think that teaching under the "two basics" principle in China means emphasizing memorization, imitation, and manipulation (Watkins and Biggs, 1996). Since the 1990s, with the effort of numerous mathematics teachers, "two basics" teaching has been raised to the level of mathematical thinking, giving rise to a series of meaningful scientific teaching methods that also match students' thinking processes.

Quality mathematics education should consist of a "solid mathematics foundation" and "progressive mathematics innovation".With modern mathematics education theories, mathematics teaching under the "two basics" principle has gradually entered a new phase. A special trend is to incorporate the "two basics" of mathematics teaching into teaching using open ended problems. In fact, open ended problem solving has become a fashion in China.







In the East, open ended problem solving was initiated in Japan in the 1970s. Then it got introduced into China. In the early 1990s, mathematics teachers used open ended problems in their classroom teaching. A key step is that open ended problems appeared in the test papers of many very significant examinations. As a teaching approach, open ended problem solving has been recommended by the national curriculum standards in 2002.

How to merge open ended problem solving with the “two basics” mathematics teaching approaches?

A classical open ended problem is the “marble problem” from Japan: “Throw five marbles into a plane, how to define the diversity of these marbles?”

This is quite an excellent problem. However, it seems far from “basic knowledge”. In the past ten years, Chinese educators designed a lot of problems in this new style. Here are some examples

Example 1. What is the common feature of the following two expressions: $8a^2b^2c^3$ and $12x^2y^3a^2$?

Possible answers are: they are both algebraic expressions; they are both single algebraic terms; the common factor in the coefficient of both the terms is 4; both contain a variable of degree 2, etc. In fact, there are many answers, but is, nevertheless, closely related to the "two basics" fundamental conceptions (Dai, 2002).

A simple open ended problem is designed to deal with number sense.

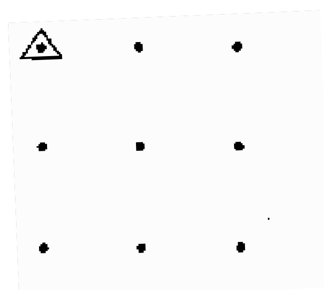
Example 2. Please insert three irrational numbers between 50π and 170.

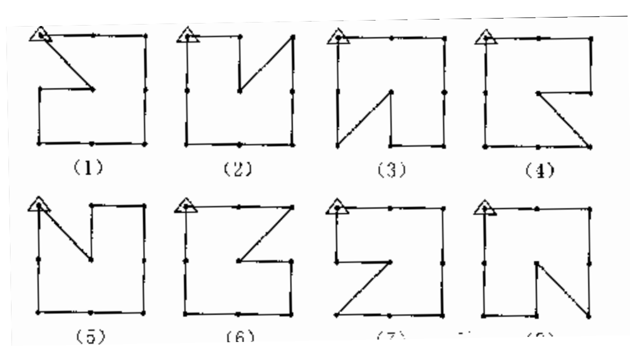
There are, of course, infinitely many correct answers. Students can use different ways to construct irrational numbers, by which they also acquire computational skill.

Another problem concerns the geometric conceptions of symmetry and reflection.

Example 3. The simple post route problem (Jinhui School, Shanghai, 1998)

There are 9 villages located in a square area (figure). The upper left corner is the post office. The postman starts from the office, covers all the 9 villages and finally returns to the office. What is the shortest route, how many routes can you find out?





1→2→4 →3 by a 90^0 rotation

1→5 by a reflection

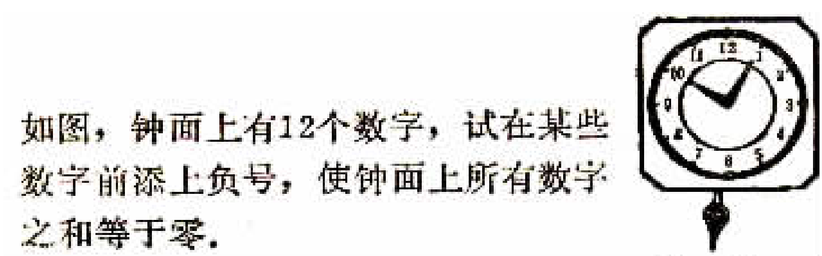
5→6→7→8 by a 90^0 rotation

This is a problem involving the combination of basic geometric knowledge and creativity.

An other interesting example is the clock-face question that occasionally appeared in the junior high school textbooks in 1993. It shows that computational skill can be combined with open ended problem solving.

Example 4. The clock-face number question

As we know there are 12 numbers on the clock-face, please add a positive or negative sign before the numbers so that their algebraic sum becomes zero.



Students can give answers using their computational skills. However, beyond the imagination of students and the authors, there are 124 correct answers! Indeed, this is quite a good open ended problem.Obviously, it is impossible for students to quote all the answers. However, they can find the general pattern that the sum of the positive numbers should be equal to that of the negative numbers. For instance, if we know that the sum of numbers with the plus (minus) symbol is 78 (–78), more answers can be found. This is closely related to the basic training sudents had on addition and subtraction of rational numbers.

Many open ended problems are just a mathematical situation. Given some conditions, but no conclusions.

Example 5. If Δ*ABC* is a triangle with right angle *C*, *CD* is perpendicular to *AB*, please find out the relations between the figures, lines, angles of the Figure A, as many as possible. This kind of problemis ppopular for use in classroom teaching in China, especially in review lessons.

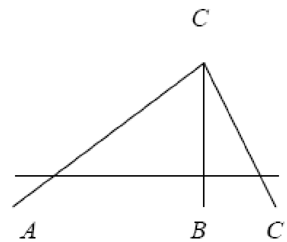
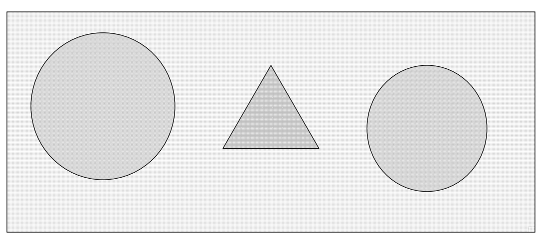


Figure A.

Some open ended problems from abroad are also in use in mathematics teaching in China, if it is sufficiently close to the “two basics”. Here is an example from Japan.

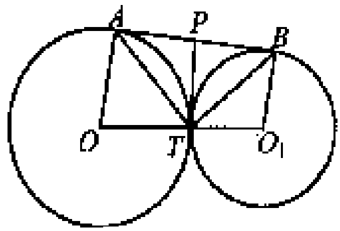
*Example 6***.** The flower bed problem (PME-13, Japan, 1993)There is a rectangular field, in which we will design a flower bed with half area of the field. Please give your various designs.



This mathematics problem is nicely related with arts, It especially relates to basic geometric knowledge and skills: figures, area, operations on real numbers, equation, square root, etc.

It is well known that examination is the bâton of mathematics education. Once open-ended problems have been introduced in the very demanding Entrance Examination papers the use of open ended problems in classroom teaching gradually became a fashion. Nowadays, every mathematics teacher in China is trying to use open ended problem in classroom teaching, and focus test questions on such problems of this style.

*Example 7***.** High school entrance exam, Hangzhou, 2001Circles O and O_1 are tangent to each other at *T*, *PT* and *AB* is the inner (exterior) common tangents respectively. Please provide a statement about this situation and its proof. (This problem is valued 12 marks. You will be evaluated according to the degree of difficulty of your conclusion.)



The examination authority establshed the following evaluation standard:

■ (6 marks) if student proved:

1. $PA = PT$ ($PB = PT$).
2. $\angle PAT = \angle PTA$ ($\angle PBT = \angle PTB$)
3. $\angle OAP = \angle OTP = 90°$.

■ (8 marks)

1. $PA = PB = PT$.
2. $\angle ATB = 90°$.
3. $\angle A0T + \angle APT = 180°$.
4. $OA \parallel O_1B$

■ (10 marks)

$$\Delta OAT \approx \Delta PTB \ (\Delta PTA \approx \Delta O_1BT)$$

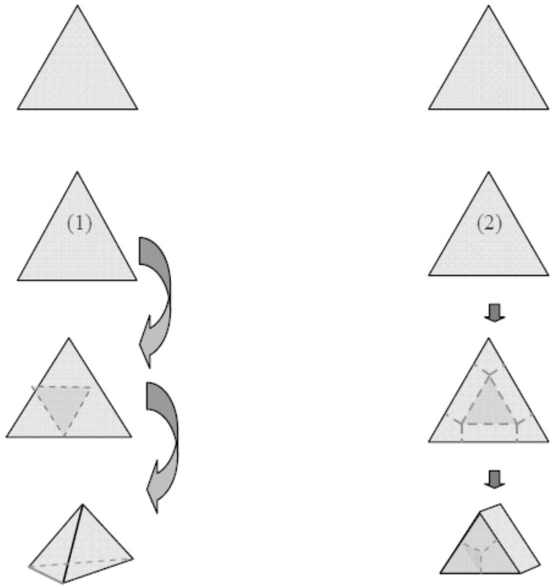
■ (12 marks)

$$PA \cdot PB = OT \cdot O_1T$$
$$(PA \cdot PB = OA \cdot O_1B)$$

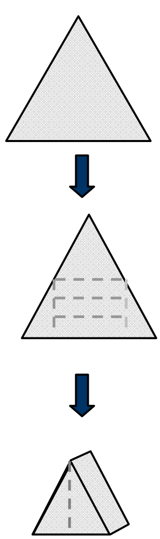
A very interesting open problem appeared on the national universities and colleges examination in 2002:

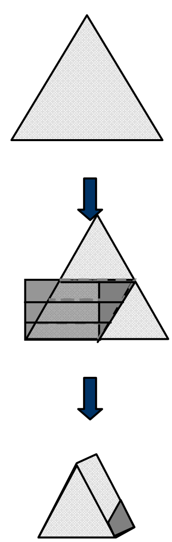
Example 8. Given two pieces of equilateral triangule shaped cardboard with the same area. Please try to:(1) Cut one paper (Figure (1)) into several pieces to form the faces of a regular tetrahedron;

(2) Cut the other paper (Figure (2)) into several pieces to form the faces of a triangular prism.



This seems to be the easiest solution of the problem. However, there are many answers. At least, the item (2) can be answered in the following two ways:





In fact, there are infinitely many answers. We may recall the “Bolyai - Gerwien Theorem” (1832):

If two simple polygons of equal area are given, one can cut the first into finitely many polygonal pieces and rearrange the pieces to obtain the second polygon. "Rearrangement" means that one may apply a translation and a rotation to every polygonal piece.

Both eastern and western mathematics education are looking for a balance between fundamentals and development (Leung, 1998; Lim, 1998). Teaching under the “two basics” in China is reaching out for new development on top of the characteristics it already possessed, striking a balance between foundations and development in classroom teaching. In any case, basics knowledge and basic skills will forever be important in one’s life. Foundations need to develop with time.

ICME 10 2004

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Regular Lecture

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Appendix

Research of rural and urban secondary students' calculation in the Jiangsu Province[4]

(1) Test subject and method

In 1998, a test was done on 2049 students from 8 schools, a total of 43 classes in the Jiangsu Province, China. The content is mathematics equations (included grouping, completing the square and factorization). 38 questions of various levels of difficulty were aksed and the time limit was 10 minutes. It was a test of students' basic mathematics skills, especially regarding the speed of completion.

(2) Test statistics

Results are shown in the following table, X is the average number of correct answers, σ_n is the standard deviation of the number of correct answers, Y(%) is the accuracy ratio. The definition of accuracy is correct answers of the tested form divided by the number of group members multiplied by the number of questions times 100%.

Grades / Variables	8th	9th	10th	11th	12th	Total
X	17.82	20.73	28.11	29.43	31.00	25.78
σ_n	6.08	7.05	4.22	4.34	4.20	7.29
Y(%)	46.89	54.55	73.97	77.45	81.58	67.84

Table 1. Average number of correct questions, standard deviation and accuracy

The testing criteria of secondary students' equation solving ability

In mathematics teaching, it is a major teaching goal to build students' basic knowledge and accompany it with solid training so as to form high standards of mathematics ability.

Based on the statistical analysis of the above samples, and combined with the experiences of mathematics teachers, 3 indices can be obtained: the average, the pass rates and the grading "excellent". According to the statistical results, the following reference standard has been compiled:

[4] Conducted by Tian Zhang, 2003.

Standard / Grades	Average	Pass	Excellent
	Correct answers	Correct answers	Correct answers
8th	17.90	≥13	≥27
9th	21.84	≥16	≥32
10th	27.67	≥24	≥34
11th	30.06	≥26	≥36
12th	31.29	≥28	≥36

Table 2. Standard reference chart for the equation solving test results

The maximum number of correct answers is 38.

Calculation test paper:

Group (I)

Solve questions 1 to 5:	Factorization (6–8):	Completing the Square (9):
1. $1 - 3x^2y + 5x^2y =$ 2. $(1/4)ab^2 - 2ab^2 =$ 3. $(1/3)xy^2 \cdot (-6x^2y) =$ 4. $6ab^2c \div (-9ac) =$ 5. $(-3xy^2)^3 =$	6. $(x2^m - 9) =$ 7. $x^2 - 3x + 2 =$ 8. $y^2 + y + (1/4) =$	9. $x^2 - 3x + 1 =$

(II) group 2

Solve questions 10 to 14:	Factorization (15–17):	Completing the square (18):
10. $-(2/3)\, ab + (3/4)\, ab + ab$ 11. $-y^2 - 2x^2 - (-3y^2)$ 12. $3x^2y \cdot (1/2)\, x \cdot (-2xy^2)$ 13. $3x^2y + (1/3)\, xy \div (-xy)$ 14. $[(-2n)^2]^3$	15. $(a + b)^2 - (x - y)^2$ 16. $(x + y)^2 + 5\,(x + y) + 6$ 17. $(m - n)^2 + 4(m - n) + 4$	18. $(1/2)\, x^2 + x + (2/3)$

(III) group 3

Solve question 19 to 20:	Factorization (21–23)	Completing the square (24)
19. $4x^3 - (-6x^3) + 9x^3$ 20. $2a^2 by\, [(-1/3)by] \div a^2 by$	21. $a^2 - ab + ac - bc$ 22. $m^2 - n^2 + am - an$ 23. $x^2 + 2xy + y^2 - z^2$	24. $x^2 + px + q$

(IV) group 4

Solve question 25 to 30:	Solve question 31 to 32:	Solve question 33 to 38:
25. $(-1/2)\, ab^2\, (b^2 + 3a^2b)$ 26. $(x - 2y^2)\, (-2x^2y)$ 27. $(m^3n + mn^2) \div (1/3)mn$ 28. $(a^3b^4c - 2ab^3c) \div (-2ab)$ 29. $(-2ab^2 + a^2b + 3ab^2)^2$ 30. $(-a^2b)^5 \div a^6b^2$	31. $(4x^2y - 5xy^2) - (3x^2y - 4xy^2)$ 32. $6ab^2\, [(-1/3)ab^4] \div 2a□(-ab^2)$	33. $2s^2t + (1/2)\, s^3t^2 \div (3/2)st + (2/3)\, s^2t$ 34. $ab^2c^2 - ab\, (1/2)\, bc^2 - ab^2c^2$ 35. $xy^2z - xy□x^2y□(-xz)$ 36. $-m^5n^3\, (1/3)\, m^2n^2mn + 4m^2$ 37. $(21a^2b^2 - 35\, a^3b^3) \div 7a^2b^2 \cdot 2ab$ 38. $2x^5y^4 \div (1/2)\, xy^2 + (-3x^2y^2)\, 2x^2$